

# THE ELK CODE MANUAL

VERSION 2.2.10



J. K. DEWHURST, S. SHARMA  
L. NORDSTRÖM, F. CRICCHIO, F. BULTMARK, O. GRÅNÄS  
E. K. U. GROSS

# Contents

<b>1</b>	<b>Introduction</b>	<b>11</b>
<b>2</b>	<b>Acknowledgments</b>	<b>11</b>
<b>3</b>	<b>Units</b>	<b>12</b>
<b>4</b>	<b>Compiling and running Elk</b>	<b>12</b>
4.1	Compiling the code . . . . .	12
4.1.1	Parallelism in Elk . . . . .	12
4.2	Linking with the libxc functional library . . . . .	14
4.3	Running the code . . . . .	14
4.3.1	Species files . . . . .	14
4.3.2	Examples . . . . .	16
<b>5</b>	<b>Input blocks</b>	<b>16</b>
5.1	atoms . . . . .	16
5.2	autokpt . . . . .	17
5.3	autolinengy . . . . .	17
5.4	autoswidth . . . . .	17
5.5	avec . . . . .	17
5.6	beta0 . . . . .	17
5.7	betamax . . . . .	18
5.8	bfieldc . . . . .	18
5.9	broydpm . . . . .	18
5.10	c_tb09 . . . . .	18
5.11	chgexs . . . . .	18
5.12	deltaem . . . . .	18
5.13	deltaph . . . . .	19
5.14	deltast . . . . .	19
5.15	dlefe . . . . .	19
5.16	dosmsum . . . . .	19
5.17	dosssum . . . . .	19
5.18	epsband . . . . .	19
5.19	epschg . . . . .	20
5.20	epsengy . . . . .	20
5.21	epsforce . . . . .	20
5.22	epslat . . . . .	20
5.23	epsocc . . . . .	20
5.24	epspt . . . . .	20
5.25	epsstress . . . . .	20
5.26	emaxelnes . . . . .	21
5.27	emaxrf . . . . .	21
5.28	fixspin . . . . .	21
5.29	fracinr . . . . .	21
5.30	fxclrc . . . . .	21
5.31	fxtyp . . . . .	21

5.32	gmaxrf	22
5.33	gmaxvr	22
5.34	hdbse	22
5.35	highq	22
5.36	hmaxvr	22
5.37	hxbse	22
5.38	hybrid	23
5.39	hybridc	23
5.40	intraband	23
5.41	isgkmax	23
5.42	kstlist	23
5.43	latvopt	23
5.44	lda+u	23
5.45	lmaxapw	24
5.46	lmaxdos	24
5.47	lmaxinr	24
5.48	lmaxmat	24
5.49	lmaxvr	24
5.50	lmirep	24
5.51	lorbcnd	25
5.52	lorbordc	25
5.53	lradstp	25
5.54	maxitoepr	25
5.55	maxscl	25
5.56	mixtype	25
5.57	mixsdb	26
5.58	mixsdpr	26
5.59	molecule	26
5.60	momfix	26
5.61	mommtfix	26
5.62	mstar	26
5.63	mustar	26
5.64	ncbse	27
5.65	ndspem	27
5.66	nempty	27
5.67	ngridk	27
5.68	ngridq	27
5.69	nosource	27
5.70	notes	28
5.71	nseqit	28
5.72	ntemp	28
5.73	nvbse	28
5.74	nwrite	28
5.75	optcomp	28
5.76	phwrite	28
5.77	plot1d	29
5.78	plot2d	29
5.79	plot3d	29

5.80	primcell	29
5.81	radkpt	29
5.82	readalu	29
5.83	reducebf	30
5.84	reduceh	30
5.85	reducek	30
5.86	reduceq	30
5.87	rgkmax	30
5.88	scale	30
5.89	scale1/2/3	31
5.90	scissor	31
5.91	scrpath	31
5.92	socscf	31
5.93	spincore	31
5.94	spinorb	31
5.95	spinpol	31
5.96	spinsprl	31
5.97	sppath	32
5.98	ssdph	32
5.99	stype	32
5.100	swidth	32
5.101	tasks	33
5.102	tau0atp	34
5.103	tau0latv	34
5.104	tauoep	35
5.105	taufsm	35
5.106	tfibs	35
5.107	tforce	35
5.108	tmomlu	35
5.109	trimvg	36
5.110	tseqit	36
5.111	tseqr	36
5.112	tshift	36
5.113	vhmat	36
5.114	vklem	36
5.115	vkloff	36
5.116	vqlss	37
5.117	wplot	37
5.118	wsfac	37
5.119	xctype	37
<b>6</b>	<b>Contributing to Elk</b>	<b>38</b>
6.1	Licensing	39
<b>7</b>	<b>Routine/Function Prologues</b>	<b>40</b>
7.1	allatoms (Source File: allatoms.f90)	40
7.2	atom (Source File: atom.f90)	40
7.3	atpstep (Source File: atpstep.f90)	41

7.4	axangsu2 (Source File: axangsu2.f90)	42
7.5	bandstr (Source File: bandstr.f90)	42
7.6	brzint (Source File: brzint.f90)	43
7.7	charge (Source File: charge.f90)	43
7.8	checkmt (Source File: checkmt.f90)	44
7.9	clebgor (Source File: clebgor.f90)	44
7.10	connect (Source File: connect.f90)	45
7.11	dielectric (Source File: dielectric.f90)	45
7.12	dos (Source File: dos.f90)	46
7.13	elfplot (Source File: elfplot.f90)	46
7.14	eliashberg (Source File: eliashberg.f90)	47
7.15	energy (Source File: energy.f90)	48
7.16	engyflu (Source File: engyflu.f90)	49
7.17	erf (Source File: erf.f90)	49
7.18	euler (Source File: euler.f90)	50
7.19	factnm (Source File: factnm.f90)	50
7.20	factr (Source File: factr.f90)	51
7.21	fderiv (Source File: fderiv.f90)	51
7.22	findband (Source File: findband.f90)	52
7.23	findlambdal (Source File: findlambdal.f90)	53
7.24	findngkmax (Source File: findngkmax.f90)	53
7.25	findprimcell (Source File: findprimcell.f90)	54
7.26	findswidth (Source File: findswidth.f90)	54
7.27	findsymcrys (Source File: findsymcrys.f90)	55
7.28	findsym (Source File: findsym.f90)	55
7.29	findsymlat (Source File: findsymlat.f90)	56
7.30	flushifc (Source File: flushifc.f90)	57
7.31	fluykwa0 (Source File: fluykwa0.f90)	57
7.32	fluykwa (Source File: fluykwa.f90)	58
7.33	force (Source File: force.f90)	58
7.34	forcek (Source File: forcek.f90)	59
7.35	fsmfield (Source File: fsmfield.f90)	60
7.36	fsmooth (Source File: fsmooth.f90)	60
7.37	gaunt (Source File: gaunt.f90)	61
7.38	gauntyry (Source File: gauntyry.f90)	61
7.39	gcd (Source File: gcd.f90)	62
7.40	genapwfr (Source File: genapwfr.f90)	62
7.41	gencfun (Source File: gencfun.f90)	63
7.42	gencore (Source File: gencore.f90)	63
7.43	genflu (Source File: genflu.f90)	64
7.44	genflufr (Source File: genflufr.f90)	64
7.45	gengkvec (Source File: gengkvec.f90)	65
7.46	gengvec (Source File: gengvec.f90)	65
7.47	genidxlo (Source File: genidxlo.f90)	66
7.48	genlofr (Source File: genlofr.f90)	67
7.49	genpmat (Source File: genpmat.f90)	67
7.50	genppts (Source File: genppts.f90)	68
7.51	genrlm (Source File: genrlm.f90)	69

7.52	genrmesh (Source File: genrmesh.f90)	70
7.53	gensdmat (Source File: gensdmat.f90)	70
7.54	gensfacgp (Source File: gensfacgp.f90)	71
7.55	genshtmat (Source File: genshtmat.f90)	71
7.56	genspchi0 (Source File: genspchi0.f90)	72
7.57	genveelu (Source File: genveelu.f90)	73
7.58	genvmatlu (Source File: genvmatlu.f90)	73
7.59	genvlijji (Source File: genvlijji.f90)	74
7.60	genvlijjk (Source File: genvlijjk.f90)	74
7.61	genvsig (Source File: genvsig.f90)	75
7.62	genwfsv (Source File: genwfsv.f90)	75
7.63	genwiq2 (Source File: genwiq2.f90)	76
7.64	genylm (Source File: genylm.f90)	77
7.65	genylmg (Source File: genylmg.f90)	77
7.66	genzrho (Source File: genzrho.f90)	78
7.67	getevecfv (Source File: getevecfv.f90)	78
7.68	getvnlijji (Source File: getvnlijji.f90)	79
7.69	getvnlijjk (Source File: getvnlijjk.f90)	79
7.70	ggair_1 (Source File: ggair_1.f90)	80
7.71	ggair_2a (Source File: ggair_2a.f90)	80
7.72	ggair_2b (Source File: ggair_2b.f90)	81
7.73	ggair_sp_1 (Source File: ggair_sp_1.f90)	81
7.74	ggair_sp_2a (Source File: ggair_sp_2a.f90)	82
7.75	ggair_sp_2b (Source File: ggair_sp_2b.f90)	82
7.76	ggamt_1 (Source File: ggamt_1.f90)	82
7.77	ggamt_2a (Source File: ggamt_2a.f90)	83
7.78	ggamt_2b (Source File: ggamt_2b.f90)	83
7.79	ggamt_sp_1 (Source File: ggamt_sp_1.f90)	84
7.80	ggamt_sp_2a (Source File: ggamt_sp_2a.f90)	84
7.81	ggamt_sp_2b (Source File: ggamt_sp_2b.f90)	85
7.82	gndstate (Source File: gndstate.f90)	86
7.83	gradrfmt (Source File: gradrfmt.f90)	86
7.84	gradzfmt (Source File: gradzfmt.f90)	87
7.85	gridsize (Source File: gridsize.f90)	88
7.86	hermite (Source File: hermite.f90)	88
7.87	hmlaa (Source File: hmlaa.f90)	89
7.88	hmlistl (Source File: hmlistl.f90)	89
7.89	hmlrad (Source File: hmlrad.f90)	90
7.90	i3mdet (Source File: i3mdet.f90)	90
7.91	i3minv (Source File: i3minv.f90)	91
7.92	i3mtv (Source File: i3mtv.f90)	91
7.93	init0 (Source File: init0.f90)	91
7.94	init1 (Source File: init1.f90)	92
7.95	linengy (Source File: linengy.f90)	92
7.96	lopzflm (Source File: lopzflm.f90)	93
7.97	match (Source File: match.f90)	93
7.98	mixadapt (Source File: mixadapt.f90)	94
7.99	mixpack (Source File: mixpack.f90)	95

7.100mixpulay (Source File: mixpulay.f90) . . . . .	96
7.101randomu (Source File: modrandom.f90) . . . . .	96
7.102xcifc (Source File: modxcifc.f90) . . . . .	97
7.103getxcdata (Source File: modxcifc.f90) . . . . .	98
7.104moment (Source File: moment.f90) . . . . .	99
7.105mossbauer (Source File: mossbauer.f90) . . . . .	99
7.106mtdmin (Source File: mtdmin.f90) . . . . .	100
7.107nfftifc (Source File: nfftifc.f90) . . . . .	100
7.108nonlinopt (Source File: nonlinopt.f90) . . . . .	101
7.109occupy (Source File: occupy.f90) . . . . .	101
7.110olpistl (Source File: olpistl.f90) . . . . .	102
7.111olprad (Source File: olprad.f90) . . . . .	102
7.112pade (Source File: pade.f90) . . . . .	103
7.113plot1d (Source File: plot1d.f90) . . . . .	103
7.114plot2d (Source File: plot2d.f90) . . . . .	104
7.115plot3d (Source File: plot3d.f90) . . . . .	104
7.116polynom (Source File: polynom.f90) . . . . .	105
7.117potcoul (Source File: potcoul.f90) . . . . .	105
7.118potks (Source File: potks.f90) . . . . .	106
7.119potplot (Source File: potplot.f90) . . . . .	106
7.120potxc (Source File: potxc.f90) . . . . .	106
7.121r3cross (Source File: r3cross.f90) . . . . .	107
7.122r3frac (Source File: r3frac.f90) . . . . .	107
7.123r3mdet (Source File: r3mdet.f90) . . . . .	108
7.124r3minv (Source File: r3minv.f90) . . . . .	108
7.125r3mm (Source File: r3mm.f90) . . . . .	108
7.126r3mmt (Source File: r3mmt.f90) . . . . .	109
7.127r3mtm (Source File: r3mtm.f90) . . . . .	109
7.128r3mtv (Source File: r3mtv.f90) . . . . .	110
7.129r3mv (Source File: r3mv.f90) . . . . .	110
7.130radnucl (Source File: radnucl.f90) . . . . .	110
7.131rdiracdme (Source File: rdiracdme.f90) . . . . .	111
7.132rdirac (Source File: rdirac.f90) . . . . .	112
7.133rdiracint (Source File: rdiracint.f90) . . . . .	112
7.134rdmdedc (Source File: rdmdedc.f90) . . . . .	113
7.135rdmdedn (Source File: rdmdedn.f90) . . . . .	114
7.136rdmdexcdc (Source File: rdmdexcdc.f90) . . . . .	114
7.137rdmdexcdn (Source File: rdmdexcdn.f90) . . . . .	115
7.138rdmdkdc (Source File: rdmdkdc.f90) . . . . .	115
7.139rdmdtsdn (Source File: rdmdtsdn.f90) . . . . .	116
7.140rdmenergy (Source File: rdmenergy.f90) . . . . .	116
7.141rdmengyxc (Source File: rdmengyxc.f90) . . . . .	117
7.142rdmentropy (Source File: rdmentropy.f90) . . . . .	117
7.143rdmeval (Source File: rdmeval.f90) . . . . .	117
7.144rdmft (Source File: rdmft.f90) . . . . .	118
7.145rdmminc (Source File: rdmminc.f90) . . . . .	118
7.146rdmminn (Source File: rdmminn.f90) . . . . .	119
7.147rdmvaryc (Source File: rdmvaryc.f90) . . . . .	119

7.148rdmvaryn (Source File: rdmvaryn.f90)	119
7.149rdmwritededn (Source File: rdmwritededn.f90)	120
7.150rdmwriteengy (Source File: rdmwriteengy.f90)	120
7.151readfermi (Source File: readfermi.f90)	121
7.152readinput (Source File: readinput.f90)	121
7.153readstate (Source File: readstate.f90)	122
7.154reciplat (Source File: recipat.f90)	122
7.155rfinp (Source File: rfinp.f90)	123
7.156rfinterp (Source File: rfinterp.f90)	123
7.157rfmtctof (Source File: rfmtctof.f90)	124
7.158rfmtinp (Source File: rfmtinp.f90)	124
7.159rhocore (Source File: rhocore.f90)	125
7.160rhoinit (Source File: rhoinit.f90)	126
7.161rhomagk (Source File: rhomagk.f90)	126
7.162rhomagsh (Source File: rhomagsh.f90)	127
7.163rhonorm (Source File: rhonorm.f90)	127
7.164rhoplot (Source File: rhoplot.f90)	127
7.165rlmrot (Source File: rlmrot.f90)	128
7.166rotaxang (Source File: rotaxang.f90)	129
7.167rot rflm (Source File: rot rflm.f90)	129
7.168rotzflm (Source File: rotzflm.f90)	130
7.169rschrodapp (Source File: rschrodapp.f90)	130
7.170rschroddme (Source File: rschroddme.f90)	131
7.171rschrodint (Source File: rschrodint.f90)	132
7.172rtozflm (Source File: rtozflm.f90)	133
7.173rvfcross (Source File: rvfcross.f90)	133
7.174sbesseldm (Source File: sbesseldm.f90)	134
7.175sbessel (Source File: sbessel.f90)	135
7.176sdelta (Source File: sdelta.f90)	135
7.177getsdata (Source File: sdelta.f90)	136
7.178sdelta_fd (Source File: sdelta_fd.f90)	136
7.179sdelta_mp (Source File: sdelta_mp.f90)	137
7.180sdelta_sq (Source File: sdelta_sq.f90)	137
7.181seceqn (Source File: seceqn.f90)	138
7.182seceqnfv (Source File: seceqnfv.f90)	138
7.183seceqnfvr (Source File: seceqnfvr.f90)	139
7.184sfacmag (Source File: sfacmag.f90)	139
7.185sfacrho (Source File: sfacrho.f90)	140
7.186sortidx (Source File: sortidx.f90)	140
7.187sphcover (Source File: sphcover.f90)	141
7.188sphcrd (Source File: sphcrd.f90)	141
7.189spline (Source File: spline.f90)	142
7.190stheta (Source File: stheta.f90)	142
7.191stheta_fd (Source File: stheta_fd.f90)	143
7.192stheta_mp (Source File: stheta_mp.f90)	143
7.193stheta_sq (Source File: stheta_sq.f90)	144
7.194sumrule (Source File: sumrule.f90)	144
7.195symrf (Source File: symrf.f90)	145

7.196	symrfir (Source File: symrfir.f90)	145
7.197	symrvf (Source File: symrvf.f90)	146
7.198	symrvfir (Source File: symrvfir.f90)	146
7.199	symvect (Source File: symvect.f90)	147
7.200	tensmomengy (Source File: tensmomengy.f90)	147
7.201	tensmom (Source File: tensmom.f90)	148
7.202	vecfbz (Source File: vecfbz.f90)	148
7.203	vecplot (Source File: vecplot.f90)	149
7.204	wavfmt (Source File: wavfmt.f90)	149
7.205	wigner3j (Source File: wigner3j.f90)	150
7.206	wigner6j (Source File: wigner6j.f90)	151
7.207	writeefg (Source File: writeefg.f90)	151
7.208	writeengyflu (Source File: writeengyflu.f90)	152
7.209	writeeval (Source File: writeeval.f90)	152
7.210	writefermi (Source File: writefermi.f90)	153
7.211	writegeom (Source File: writegeom.f90)	153
7.212	writeiad (Source File: writeiad.f90)	153
7.213	writeinfo (Source File: writeinfo.f90)	154
7.214	writekpts (Source File: writekpts.f90)	154
7.215	writelinen (Source File: writelinen.f90)	155
7.216	writemat (Source File: writemat.f90)	155
7.217	writestate (Source File: writestate.f90)	156
7.218	writesym (Source File: writesym.f90)	156
7.219	writevnljji (Source File: writevnljji.f90)	156
7.220	writevnljjk (Source File: writevnljjk.f90)	157
7.221	writewiq2 (Source File: writewiq2.f90)	157
7.222	xc_am05 (Source File: xc_am05.f90)	158
7.223	xc_am05_point (Source File: xc_am05.f90)	158
7.224	xc_am05_ldax (Source File: xc_am05.f90)	159
7.225	xc_am05_ldapwc (Source File: xc_am05.f90)	159
7.226	xc_am05_labertw (Source File: xc_am05.f90)	159
7.227	xc_pbe (Source File: xc_pbe.f90)	160
7.228	xc_pwca (Source File: xc_pwca.f90)	161
7.229	xc_pzca (Source File: xc_pzca.f90)	161
7.230	xc_vbh (Source File: xc_vbh.f90)	162
7.231	xc_xalpha (Source File: xc_xalpha.f90)	163
7.232	ylmrot (Source File: ylmrot.f90)	163
7.233	ylmroty (Source File: ylmroty.f90)	164
7.234	z2mctm (Source File: z2mctm.f90)	164
7.235	z2mmct (Source File: z2mmct.f90)	165
7.236	z2mm (Source File: z2mm.f90)	165
7.237	zbessela (Source File: zbessela.f90)	165
7.238	zbesselb (Source File: zbesselb.f90)	166
7.239	zfftifc (Source File: zfftifc.f90)	167
7.240	zfinp (Source File: zfinp.f90)	167
7.241	zflmconj (Source File: zflmconj.f90)	168
7.242	zfmtinp (Source File: zfmtinp.f90)	169
7.243	zfrf (Source File: zfrf.f90)	169

7.244zpotclmt (Source File: zpotclmt.f90) . . . . .	170
7.245zpotcoul (Source File: zpotcoul.f90) . . . . .	171
7.246ztorflm (Source File: ztorflm.f90) . . . . .	173

## 1 Introduction

Welcome to the Elk Code! Elk is an all-electron full-potential linearised augmented-plane-wave (FP-LAPW) code for determining the properties of crystalline solids. It was developed originally at the Karl-Franzens-Universität Graz as part of the EXCITING EU Research and Training Network project<sup>1</sup>. The guiding philosophy during the implementation of the code was to keep it as simple as possible for both users and developers without compromising on its capabilities. All the routines are released under either the GNU General Public License (GPL) or the GNU Lesser General Public License (LGPL) in the hope that they may inspire other scientists to implement new developments in the field of density functional theory and beyond.

## 2 Acknowledgments

Lots of people contributed to the Elk code with ideas, checking and testing, writing code or documentation and general encouragement. They include Claudia Ambrosch-Draxl, Clas Persson, Christian Brouder, Rickard Armiento, Andrew Chizmeshya, Per Anderson, Igor Nekrasov, Sushil Auluck, Frank Wagner, Fateh Kalarasse, Jürgen Spitaler, Stefano Pittalis, Nektarios Lathiotakis, Tobias Burnus, Stephan Sagmeister, Christian Meisenbichler, Sébastien Lebègue, Yigang Zhang, Fritz Körmann, Alexey Baranov, Anton Kozhevnikov, Shigeru Suehara, Frank Essenberger, Antonio Sanna, Tyrel McQueen, Tim Baldsiefen, Marty Blaber, Anton Filanovich, Torbjörn Björkman, Martin Stankovski, Jerzy Goraus, Markus Meinert, Daniel Rohr, Vladimir Nazarov, Kevin Krieger, Pink Floyd, Arkardy Davydov, Florian Eich and Aldo Romero Castro. Special mention of David Singh's very useful book on the LAPW method<sup>2</sup> must also be made. Finally we would like to acknowledge the generous support of Karl-Franzens-Universität Graz, as well as the EU Marie-Curie Research Training Networks initiative.

Kay Dewhurst  
Sangeeta Sharma  
Lars Nordström  
Francesco Cricchio  
Fredrik Bultmark  
Oscar Grånäs  
Hardy Gross

Halle and Uppsala, December 2013

---

<sup>1</sup>EXCITING code developed under the Research and Training Network EXCITING funded by the EU, contract No. HPRN-CT-2002-00317

<sup>2</sup> D. J. Singh, *Planewaves, Pseudopotentials and the LAPW Method* (Kluwer Academic Publishers, Boston, 1994).

## 3 Units

Unless explicitly stated otherwise, Elk uses atomic units. In this system  $\hbar = 1$ , the electron mass  $m = 1$ , the Bohr radius  $a_0 = 1$  and the electron charge  $e = 1$  (note that the electron charge is positive, so that the atomic numbers  $Z$  are negative). Thus, the atomic unit of length is 0.52917720859(36) Å, and the atomic unit of energy is the Hartree which equals 27.21138386(68) eV. The unit of the external magnetic fields is defined such that one unit of magnetic field in `elk.in` equals 1715.255397557 Tesla.

## 4 Compiling and running Elk

### 4.1 Compiling the code

Unpack the code from the archive file. Run the command

```
setup
```

in the `elk` directory and select the appropriate system and compiler. We highly recommend that you edit the file `make.inc` and tune the compiler options for your computer system. In particular, use of machine-optimised BLAS/LAPACK libraries can result in significant increase in performance, but make sure they are of version 3.x. Following this, run

```
make
```

This will hopefully compile the entire code and all the libraries into one executable, `elk`, located in the `elk/src` directory. It will also compile two useful auxiliary programs, namely `spacegroup` for producing crystal geometries from spacegroup data and `eos` for fitting equations of state to energy-volume data. If you want to compile everything all over again, then run `make clean` from the `elk` directory, followed by `make`.

#### 4.1.1 Parallelism in Elk

Three forms of parallelism are implemented in Elk, and all can be used in combination with each other, with efficiency depending on the particular task, crystal structure and computer system. You may need to contact your system administrator for assistance with running Elk in parallel.

1. OpenMP works for symmetric multiprocessors, i.e. computers that have many cores with the same unified memory accessible to each. It is enabled by setting the appropriate command-line options (e.g. `-openmp` for the Intel compiler) before compiling, and also at runtime by the environment variable

```
export OMP_NUM_THREADS=x
```

where `x` is the number of cores available on a particular node. Elk also makes use of nested OpenMP parallelism, meaning that there are nested parallel loops in parts of the code, the iterative diagonaliser being one such example. These nested loops can be run in parallel by setting

```
export OMP_NESTED=true
```

Be aware that using this option can actually degrade performance if too many threads are created. To avoid such oversubscription, you may need to adjust related OpenMP variables including `OMP_THREAD_LIMIT`, `OMP_MAX_ACTIVE_LEVELS` and `OMP_DYNAMIC` (there may also be additional compiler-specific environment variables that can be set). Finally, some vendor-supplied BLAS/LAPACK libraries use OpenMP internally, for example MKL from Intel and ACML from AMD; refer to their documentation for usage.

2. The message passing interface (MPI) is particularly suitable for running Elk across multiple nodes of a cluster, with scaling to hundreds of processors possible. To enable MPI, comment out the indicated line in `elk/src/Makefile` and set the variable `F90` in `elk/make.inc` to the MPI version of Fortran 90 (usually `mpif90`). Then run `make clean` followed by `make`. If  $y$  is the number of nodes and  $x$  is the number of cores per node, then at runtime invoke

```
mpirun -np z ./elk
```

where  $z = x \cdot y$  is the total number of cores available on the machine. Highest efficiency is obtained by using hybrid parallelism with OpenMP on each node and MPI across nodes. This can be done by compiling the code using the MPI Fortran compiler in combination with the OpenMP command-line option. At runtime set `export OMP_NUM_THREADS=x` and start the MPI run with *one process per node* as follows

```
mpirun -pernode -np y ./elk
```

The number of MPI processes is reported in the file `INFO.OUT` which serves as a check that MPI is running correctly. Note that version 2 of the MPI libraries is required to run Elk.

3. Phonon calculations use a simple form of parallelism by just examining the run directory for dynamical matrix files. These files are of the form

```
DYN_Qqqqq_qqqq_qqqq_Sss_Aaa_Pp.OUT
```

and contain a single row of a particular dynamical matrix. Elk simply finds which `DYN` files do not exist, chooses one and runs it. This way many independent runs of Elk can be started in the same directory on a networked file system (NFS), and will run until all the dynamical matrices files are completed. Should a particular run crash, then delete the associated empty `DYN` file and rerun Elk.

## 4.2 Linking with the libxc functional library

Libxc is the ETSF library of exchange-correlation functionals. Elk can use the complete set of LDA and GGA functionals available in libxc. In order to do this, first download and compile libxc. This should have produced the file `libxc.a`. Copy this file to the `elk/src` directory and then uncomment the lines indicated for libxc in the file `elk/src/Makefile`. Once this is done, run `make clean` followed by `make`. To select a particular functional of libxc, use the block

```
xctype
  100 nx nc
```

where `nx` and `nc` are, respectively, the numbers of the exchange and correlation functionals in the libxc library. See the file `elk/src/libxc_funcs.f90` for a list of the functionals and their associated numbers.

## 4.3 Running the code

As a rule, all input files for the code are in lower case and end with the extension `.in`. All output files are uppercase and have the extension `.OUT`. For most cases, the user will only need to modify the file `elk.in`. In this file input parameters are arranged in blocks. Each block consists of a block name on one line and the block variables on subsequent lines. Almost all blocks are optional: the code uses reasonable default values in cases where they are absent. Blocks can appear in any order, if a block is repeated then the second instance is used. Comment lines can be included in the input file and begin with the `!` character.

### 4.3.1 Species files

The only other input files are those describing the atomic species which go into the crystal. These files are found in the `species` directory and are named with the element symbol and the extension `.in`, for example `Sb.in`. They contain parameters like the atomic charge, mass, muffin-tin radius, occupied atomic states and the type of linearisation required. Here as an example is the copper species file `Cu.in`:

```
'Cu'           : spsymb
'copper'       : spname
-29.0000       : spzn
115837.2716    : spmass
0.371391E-06   2.0000  34.8965  500 : sprmin, rmt, sprmax, nrmt
10            : spnst
1  0  1  2.00000  T : spn, spl, spk, spocc, spcore
2  0  1  2.00000  T
2  1  1  2.00000  T
2  1  2  4.00000  T
3  0  1  2.00000  T
3  1  1  2.00000  F
3  1  2  4.00000  F
3  2  2  4.00000  F
3  2  3  6.00000  F
```

4	0	1	1.00000	F	
1					: apword
0.1500	0	F			: apwe0, apwdm, apwve
1					: nlx
2	2				: lx, apword
0.1500	0	T			: apwe0, apwdm, apwve
0.1500	1	T			
4					: nlorb
0	2				: lorbl, lorbord
0.1500	0	F			: lorbe0, lorbdm, lorbve
0.1500	1	F			
1	2				
0.1500	0	F			
0.1500	1	F			
2	2				
0.1500	0	F			
0.1500	1	F			
1	3				
0.1500	0	F			
0.1500	1	F			
-2.8652	0	T			

The input parameters are defined as follows:

**spsymb**

The symbol of the element.

**spname**

The name of the element.

**spzn**

Nuclear charge: should be negative since the electron charge is taken to be positive in the code; it can also be fractional for purposes of doping.

**spmash**

Nuclear mass in atomic units.

**sprmin, rmt, sprmax, nrmt**

Respectively, the minimum radius on logarithmic radial mesh; muffin-tin radius; effective infinity for atomic radial mesh; and number of radial mesh points to muffin-tin radius.

**spnst**

Number of atomic states.

**spn, spl, spk, spocc, spcore**

Respectively, the principal quantum number of the radial Dirac equation; quantum number  $l$ ; quantum number  $k$  ( $l$  or  $l + 1$ ); occupancy of atomic state (can be fractional); .T. if state is in the core and therefore treated with the Dirac equation in the spherical part of the muffin-tin Kohn-Sham potential.

**apword**

Default APW function order, i.e. the number of radial functions and therefore the order of

the radial derivative matching at the muffin-tin surface.

`apwe0`, `apwdm`, `apwve`

Respectively, the default APW linearisation energy; the order of the energy derivative of the APW radial function  $\partial^m u(r)/\partial E^m$ ; and `.T.` if the linearisation energy is allowed to vary.

`n1x`

The number of exceptions to the default APW configuration. These should be listed on subsequent lines for particular angular momenta. In this example, the fixed energy APW with angular momentum  $d$  (`1x = 2`) is replaced with a LAPW, which has variable linearisation energy.

`nlorb`

Number of local orbitals.

`lorbl`, `lorbord`

Respectively, the angular momentum  $l$  of the local-orbital; and the order of the radial derivative which goes to zero at the muffin-tin surface.

`lorbe0`, `lorbdm`, `lorbve`

Respectively, the default local-orbital linearisation energy; the order of the energy derivative of the local-orbital radial function; and `.T.` if the linearisation energy is allowed to vary.

### 4.3.2 Examples

The best way to learn to use Elk is to run the examples included with the package. These can be found in the `examples` directory and use many of the code's capabilities. The following section which describes all the input parameters will be of invaluable assistance.

## 5 Input blocks

This section lists all the input blocks available. It is arranged with the name of the block followed by a table which lists each parameter name, what the parameter does, its type and default value. A horizontal line in the table indicates a new line in `elk.in`. Below the table is a brief overview of the block's function.

### 5.1 atoms

<code>nspecies</code>	number of species	integer	0
<code>spfname(i)</code>	species filename for species $i$	string	-
<code>natoms(i)</code>	number of atoms for species $i$	integer	-
<code>atposl(j,i)</code>	atomic position in lattice coordinates for atom $j$	real(3)	-
<code>bfcmt(j,i)</code>	muffin-tin external magnetic field in Cartesian coordinates for atom $j$	real(3)	-

Defines the atomic species as well as their positions in the unit cell and the external magnetic field applied throughout the muffin-tin. These fields are used to break spin symmetry and should be considered infinitesimal as they do not contribute directly to the total energy. Collinear calculations are more efficient if the field is applied in the  $z$ -direction. One could, for example, set up an anti-ferromagnetic crystal by pointing the field on one atom in the

positive  $z$ -direction and in the opposite direction on another atom. If `molecule` is `.true.` then the atomic positions are assumed to be in Cartesian coordinates. See also `sppath`, `bfieldc` and `molecule`.

## 5.2 autokpt

<code>autokpt</code>	<code>.true.</code> if the $k$ -point set is to be determined automatically	logical	<code>.false.</code>
----------------------	---	---------	----------------------

See `radkpt` for details.

## 5.3 autolinengy

<code>autolinengy</code>	<code>.true.</code> if the fixed linearisation energies are to be determined automatically	logical	<code>.false.</code>
--------------------------	--	---------	----------------------

See `dlefe` for details.

## 5.4 autoswidth

<code>autoswidth</code>	<code>.true.</code> if the smearing parameter <code>swidth</code> should be determined automatically	logical	<code>.false.</code>
-------------------------	--	---------	----------------------

Calculates the smearing width from the  $k$ -point density,  $V_{\text{BZ}}/n_k$ ; the valence band width,  $W$ ; and an effective mass parameter,  $m^*$ ; according to

$$\sigma = \frac{\sqrt{2W}}{m^*} \left( \frac{3}{4\pi} \frac{V_{\text{BZ}}}{n_k} \right)^{1/3}.$$

The variable `mstar` then replaces `swidth` as the control parameter of the smearing width. A large value of  $m^*$  gives a narrower smearing function. Since `swidth` is adjusted according to the fineness of the  $\mathbf{k}$ -mesh, the smearing parameter can then be eliminated. It is not recommended that `autoswidth` be used in conjunction with the Fermi-Dirac smearing function, since the electronic temperature will then be a function of the  $k$ -point mesh. See T. Björkman and O. Grånäs, *Int. J. Quant. Chem.* DOI: 10.1002/qua.22476 (2010) for details. See also `stype` and `swidth`.

## 5.5 avec

<code>avec(1)</code>	first lattice vector	real(3)	(1.0, 0.0, 0.0)
<code>avec(2)</code>	second lattice vector	real(3)	(0.0, 1.0, 0.0)
<code>avec(3)</code>	third lattice vector	real(3)	(0.0, 0.0, 1.0)

Lattice vectors of the crystal in atomic units (Bohr).

## 5.6 beta0

<code>beta0</code>	adaptive mixing parameter	real	0.05
--------------------	---------------------------	------	------

This determines how much of the potential from the previous self-consistent loop is mixed with the potential from the current loop. It should be made smaller if the calculation is unstable. See `betamax` and also the routine `mixadapt`.

## 5.7 betamax

betamax	maximum adaptive mixing parameter	real	0.5
---------	-----------------------------------	------	-----

Maximum allowed mixing parameter used in routine `mixadapt`.

## 5.8 bfieldc

bfieldc	global external magnetic field in Cartesian coordinates	real(3)	(0.0,0.0,0.0)
---------	---	---------	---------------

This is a constant magnetic field applied throughout the entire unit cell and enters the second-variational Hamiltonian as

$$\frac{g_e}{4c} \vec{\sigma} \cdot \mathbf{B}_{\text{ext}},$$

where  $g_e$  is the electron  $g$ -factor. This field is normally used to break spin symmetry for spin-polarised calculations and considered to be infinitesimal with no direct contribution to the total energy. In cases where the magnetic field is finite (for example when computing magnetic response) the external  $\mathbf{B}$ -field energy reported in `INFO.OUT` should be added to the total by hand. This field is applied throughout the entire unit cell. To apply magnetic fields in particular muffin-tins use the `bfcmt` vectors in the `atoms` block. Collinear calculations are more efficient if the field is applied in the  $z$ -direction.

## 5.9 broydpn

broydpn	Broyden mixing parameters $\alpha$ and $w_0$	real	(0.25,0.01)
---------	--	------	-------------

See `mixtype` and `mixsdb`.

## 5.10 c\_tb09

c_tb09	Tran-Blaha constant $c$	real	-
--------	-------------------------	------	---

Sets the constant  $c$  in the Tran-Blaha '09 functional. Normally this is calculated from the density, but there may be situations where this needs to be adjusted by hand. See *Phys. Rev. Lett.* **102**, 226401 (2009).

## 5.11 chgexs

chgexs	excess electronic charge	real	0.0
--------	--------------------------	------	-----

This controls the amount of charge in the unit cell beyond that required to maintain neutrality. It can be set positive or negative depending on whether electron or hole doping is required.

## 5.12 deltaem

deltaem	the size of the $\mathbf{k}$ -vector displacement used when calculating numerical derivatives for the effective mass tensor	real	0.025
---------	---	------	-------

See `ndspem` and `vklem`.

### 5.13 `deltaph`

<code>deltaph</code>	size of the atomic displacement used for calculating dynamical matrices	real	0.02
----------------------	---	------	------

Phonon calculations are performed by constructing a supercell corresponding to a particular  $\mathbf{q}$ -vector and making a small periodic displacement of the atoms. The magnitude of this displacement is given by `deltaph`. This should not be made too large, as anharmonic terms could then become significant, neither should it be too small as this can introduce numerical error.

### 5.14 `deltast`

<code>deltast</code>	size of the change in lattice vectors used for calculating the stress tensor	real	0.01
----------------------	--	------	------

The stress tensor is computed by changing the lattice vector matrix  $A$  by

$$A \rightarrow (1 + \delta t e_i)A,$$

where  $dt$  is an infinitesimal equal in practice to `deltast` and  $e_i$  is the  $i^{\text{th}}$  strain tensor. Numerical finite differences are used to compute the stress tensor as the derivative of the total energy  $dE_i/dt$ .

### 5.15 `dlefe`

<code>dlefe</code>	difference between the fixed linearisation energy and the Fermi energy	real	-0.1
--------------------	--	------	------

When `autolinergy` is `.true.` then the fixed linearisation energies are set to the Fermi energy plus `dlefe`.

### 5.16 `dosmsum`

<code>dosmsum</code>	<code>.true.</code> if the partial DOS is to be summed over $m$	logical	<code>.false.</code>
----------------------	---	---------	----------------------

By default, the partial density of states is resolved over  $(l, m)$  quantum numbers. If `dosmsum` is set to `.true.` then the partial DOS is summed over  $m$ , and thus depends only on  $l$ .

### 5.17 `dosssum`

<code>dosssum</code>	<code>.true.</code> if the partial DOS is to be summed over spin	logical	<code>.false.</code>
----------------------	--	---------	----------------------

By default, the partial density of states for spin-polarised systems is spin resolved.

### 5.18 `epsband`

<code>epsband</code>	convergence tolerance for determining band energies	real	$1 \times 10^{-12}$
----------------------	---	------	---------------------

APW and local-orbital linearisation energies are determined from the band energies. This is done by first searching upwards in energy until the radial wavefunction at the muffin-tin radius is zero. This is the energy at the top of the band, denoted  $E_t$ . A downward search is now performed from  $E_t$  until the slope of the radial wavefunction at the muffin-tin radius is zero. This energy,  $E_b$ , is at the bottom of the band. The band energy is taken

as  $(E_t + E_b)/2$ . If either  $E_t$  or  $E_b$  is not found, then the band energy is set to the default value.

### 5.19 epschg

<b>epschg</b>	maximum allowed error in the calculated total charge beyond which a warning message will be issued	real	$1 \times 10^{-3}$
---------------	--	------	--------------------

### 5.20 epsengy

<b>epsengy</b>	convergence criterion for the total energy	real	$1 \times 10^{-4}$
----------------	--	------	--------------------

See **epspt**.

### 5.21 epsforce

<b>epsforce</b>	convergence tolerance for the forces during a geometry optimisation run	real	$5 \times 10^{-4}$
-----------------	---	------	--------------------

If the mean absolute value of the atomic forces is less than **epsforce** then the geometry optimisation run is ended. See also **tasks** and **latvopt**.

### 5.22 epslat

<b>epslat</b>	vectors with lengths less than this are considered zero	real	$10^{-6}$
---------------	---	------	-----------

Sets the tolerance for determining if a vector or its components are zero. This is to account for any numerical error in real or reciprocal space vectors.

### 5.23 epsocc

<b>epsocc</b>	smallest occupancy for which a state will contribute to the density	real	$1 \times 10^{-8}$
---------------	---	------	--------------------

### 5.24 epspt

<b>epspt</b>	convergence criterion for the Kohn-Sham potential and field	real	$1 \times 10^{-6}$
--------------	---	------	--------------------

If the RMS change in the Kohn-Sham potential and magnetic field is smaller than **epspt** and the absolute change in the total energy is less than **epsengy**, then the self-consistent loop is considered converged and exited. For geometry optimisation runs this results in the forces being calculated, the atomic positions updated and the loop restarted. See also **epsengy** and **maxscl**.

### 5.25 epsstress

<b>epsstress</b>	convergence tolerance for the stress tensor during a geometry optimisation run with lattice vector relaxation	real	$5 \times 10^{-3}$
------------------	---	------	--------------------

See also **epsforce** and **latvopt**.

### 5.26 emaxelnes

<b>emaxelnes</b>	maximum allowed initial-state eigenvalue for ELNES calculations	real	-1.2
------------------	---	------	------

### 5.27 emaxrf

<b>emaxrf</b>	energy cut-off used when calculating Kohn-Sham response functions	real	$10^6$
---------------	---	------	--------

A typical Kohn-Sham response function is of the form

$$\chi_s(\mathbf{r}, \mathbf{r}', \omega) \equiv \frac{\delta\rho(\mathbf{r}, \omega)}{\delta v_s(\mathbf{r}', \omega)} = \frac{1}{N_k} \sum_{i\mathbf{k}, j\mathbf{k}'} (f_{i\mathbf{k}} - f_{j\mathbf{k}'}) \frac{\langle i\mathbf{k} | \hat{\rho}(\mathbf{r}) | j\mathbf{k}' \rangle \langle j\mathbf{k}' | \hat{\rho}(\mathbf{r}') | i\mathbf{k} \rangle}{\omega + (\varepsilon_{i\mathbf{k}} - \varepsilon_{j\mathbf{k}'}) + i\eta},$$

where  $\hat{\rho}$  is the density operator;  $N_k$  is the number of  $k$ -points;  $\varepsilon_{i\mathbf{k}}$  and  $f_{i\mathbf{k}}$  are the eigenvalues and occupation numbers, respectively. The variable **emaxrf** is an energy window which limits the summation over states in the formula above so that  $|\varepsilon_{i\mathbf{k}} - \varepsilon_{\text{Fermi}}| < \mathbf{emaxrf}$ . Reducing this can result in a faster calculation at the expense of accuracy.

### 5.28 fixspin

<b>fixspin</b>	0 for no fixed spin moment (FSM), 1 for total FSM, 2 for local muffin-tin FSM, and 3 for both total and local FSM	integer	0
----------------	---	---------	---

Set to 1, 2 or 3 for fixed spin moment calculations. To fix only the direction and not the magnitude set to  $-1$ ,  $-2$  or  $-3$ . See also **momfix**, **mommtfix**, **taufsm** and **spinpol**.

### 5.29 fracinr

<b>fracinr</b>	fraction of the muffin-tin radius up to which <b>lmaxinr</b> is used as the angular momentum cut-off	real	0.25
----------------	--	------	------

See **lmaxinr**.

### 5.30 fxclrc

<b>fxclrc</b>	parameters for the dynamical long-range contribution (LRC) to the TDDFT exchange-correlation kernel	real(2)	(0.0, 0.0)
---------------	---	---------	------------

These are the parameters  $\alpha$  and  $\beta$  for the kernel proposed in *Phys. Rev. B* **72**, 125203 (2005), namely

$$f_{xc}(\mathbf{G}, \mathbf{G}', \mathbf{q}, \omega) = -\frac{\alpha + \beta\omega^2}{q^2} \delta_{\mathbf{G}, \mathbf{G}'} \delta_{\mathbf{G}, \mathbf{0}}.$$

### 5.31 fxctype

<b>fxctype</b>	integer defining the type of exchange-correlation kernel $f_{xc}$	integer	-1
----------------	---	---------	----

The acceptable values are:

- 1  $f_{xc}$  defined by `xctype`
- 0,1 RPA ( $f_{xc} = 0$ )
- 200 Long-range contribution (LRC) kernel, S. Botti *et al.*, *Phys. Rev. B* **72**, 125203 (2005); see `fxclrc`
- 210 ‘Bootstrap’ kernel, S. Sharma, J. K. Dewhurst, A. Sanna and E. K. U. Gross, *Phys. Rev. Lett.* **107**, 186401 (2011)

### 5.32 `gmaxrf`

<code>gmaxrf</code>	maximum length of $ \mathbf{G} $ for computing response functions	real	3.0
---------------------	---	------	-----

### 5.33 `gmaxvr`

<code>gmaxvr</code>	maximum length of $ \mathbf{G} $ for expanding the interstitial density and potential	real	12.0
---------------------	---	------	------

This variable has a lower bound which is enforced by the code as follows:

$$\text{gmaxvr} \rightarrow \max(\text{gmaxvr}, 2 \times \text{gkmax} + \text{epslat})$$

See `rgkmax` and `trimvg`.

### 5.34 `hdbse`

<code>hdbse</code>	<code>.true.</code> if the direct term is to be included in the BSE Hamiltonian	logical	<code>.true.</code>
--------------------	---	---------	---------------------

### 5.35 `highq`

<code>highq</code>	<code>.true.</code> if a high quality parameter set should be used	logical	<code>.false.</code>
--------------------	--	---------	----------------------

Setting this to `.true.` results in some default parameters being changed to ensure good convergence in most situations. See the subroutine `readinput` for the list of changed parameters and their values. These changes can be overruled by subsequent blocks in the input file.

### 5.36 `hmaxvr`

<code>hmaxvr</code>	maximum length of $\mathbf{H}$ -vectors	real	6.0
---------------------	---	------	-----

The  $\mathbf{H}$ -vectors are used for calculating X-ray and magnetic structure factors. They are also used in linear response phonon calculations for expanding the density and potential in plane waves. See also `gmaxvr`, `vhmat`, `reduceh`, `wsfac` and `hkmax`.

### 5.37 `hxbse`

<code>hxbse</code>	<code>.true.</code> if the exchange term is to be included in the BSE Hamiltonian		<code>.true.</code>
--------------------	---	--	---------------------

### 5.38 hybrid

hybrid	.true if a hybrid functional is to be used when running a Hartree-Fock calculation	logical	.false
--------	--	---------	--------

See also `hybridc` and `xctype`.

### 5.39 hybridc

hybridc	hybrid functional mixing coefficient	real	1.0
---------	--------------------------------------	------	-----

### 5.40 intraband

intraband	.true. if the intraband (Drude-like) contribution is to be added to the dielectric tensor	logical	.false.
-----------	---	---------	---------

### 5.41 isgkmax

isgkmax	species for which the muffin-tin radius will be used for calculating <code>gkmax</code>	integer	-1
---------	---	---------	----

By default (-1) the average muffin-tin radius is used for determining `gkmax` from `rgkmax`. This can be changed by setting `isgkmax` either to the desired species number; or to -2 if `gkmax = rgkmax/2`; or to -3 in which case the smallest radius is used.

### 5.42 kstlist

kstlist(i)	<i>i</i> th <i>k</i> -point and state pair	integer(2)	(1,1)
------------	--	------------	-------

This is a user-defined list of *k*-point and state index pairs which are those used for plotting wavefunctions and writing **L**, **S** and **J** expectation values. Only the first pair is used by the aforementioned tasks. The list should be terminated by a blank line.

### 5.43 latvopt

latvopt	type of lattice vector optimisation to be performed during structural relaxation	integer	0
---------	--	---------	---

Unconstrained optimisation of the lattice vectors will be performed with `task = 2, 3` when `latvopt = 1`. When `latvopt = 2` the lattice vectors will be optimised while keeping the unit cell volume fixed. In either case, the crystal symmetry is retained during optimisation. By default (`latvopt = 0`) no lattice vector optimisation is performed during structural relaxation. See also `tau0latv`.

### 5.44 lda+u

ldapu	type of LDA+ <i>U</i> calculation	integer	0
inptypelu	type of input for LDA+ <i>U</i> calculation	integer	1
is	species number	integer	-
l	angular momentum value	integer	-1
u	the desired <i>U</i> value	real	0.0
j	the desired <i>J</i> value	real	0.0

This block contains the parameters required for an LDA+ $U$  calculation, with the list of parameters for each species terminated with a blank line. The type of double counting required is set with the parameter `ldapu`. Currently implemented are:

- 0 No LDA+ $U$  calculation
- 1 Fully localised limit (FLL)
- 2 Around mean field (AFM)
- 3 An interpolation between FLL and AFM

The type of input parameters is set with the parameter `inptypelu`. The current possibilities are:

- 1 U and J
- 2 Slater parameters
- 3 Racah parameters
- 4 Yukawa screening length
- 5 U and determination of corresponding Yukawa screening length

See (amongst others) *Phys. Rev. B* **67**, 153106 (2003), *Phys. Rev. B* **52**, R5467 (1995), *Phys. Rev. B* **60**, 10673 (1999), and *Phys. Rev. B* **80**, 035121 (2009).

#### 5.45 `lmaxapw`

<code>lmaxapw</code>	angular momentum cut-off for the APW functions	integer	8
----------------------	--	---------	---

#### 5.46 `lmaxdos`

<code>lmaxdos</code>	angular momentum cut-off for the partial DOS plot	integer	3
----------------------	---	---------	---

#### 5.47 `lmaxinr`

<code>lmaxinr</code>	angular momentum cut-off for the muffin-tin density and potential on the inner part of the muffin-tin	integer	2
----------------------	---	---------	---

Close to the nucleus, the density and potential is almost spherical and therefore the spherical harmonic expansion can be truncated a low angular momentum. See also `fracinr`.

#### 5.48 `lmaxmat`

<code>lmaxmat</code>	angular momentum cut-off for the outer-most loop in the hamiltonian and overlap matrix setup	integer	5
----------------------	--	---------	---

#### 5.49 `lmaxvr`

<code>lmaxvr</code>	angular momentum cut-off for the muffin-tin density and potential	integer	7
---------------------	---	---------	---

#### 5.50 `lmirep`

<code>lmirep</code>	<code>.true.</code> if the $Y_{lm}$ basis is to be transformed into the basis of irreducible representations of the site symmetries for DOS plotting	logical	<code>.true.</code>
---------------------	--	---------	---------------------

When `lmirep` is set to `.true.`, the spherical harmonic basis is transformed into one in which the site symmetries are block diagonal. Band characters determined from the density matrix expressed in this basis correspond to irreducible representations, and allow the partial DOS to be resolved into physically relevant contributions, for example  $e_g$  and  $t_{2g}$ .

### 5.51 `lorbcnd`

<code>lorbcnd</code>	<code>.true.</code> if conduction state local-orbitals are to be automatically added to the basis	logical	<code>.false.</code>
----------------------	---	---------	----------------------

Adding these higher energy local-orbitals can improve calculations which rely on accurate unoccupied states, such as the response function. See also `lorbordc`.

### 5.52 `lorbordc`

<code>lorbordc</code>	the order of the conduction state local-orbitals	integer	3
-----------------------	--	---------	---

See `lorbcnd`.

### 5.53 `lradstp`

<code>lradstp</code>	radial step length for determining coarse radial mesh	integer	4
----------------------	---	---------	---

Some muffin-tin functions (such as the density) are calculated on a coarse radial mesh and then interpolated onto a fine mesh. This is done for the sake of efficiency. `lradstp` defines the step size in going from the fine to the coarse radial mesh. If it is too large, loss of precision may occur.

### 5.54 `maxitoe`

<code>maxitoe</code>	maximum number of iterations when solving the exact exchange integral equations	integer	200
----------------------	---	---------	-----

See `tau0oep`.

### 5.55 `maxscl`

<code>maxscl</code>	maximum number of self-consistent loops allowed	integer	200
---------------------	---	---------	-----

This determines after how many loops the self-consistent cycle will terminate if the convergence criterion is not met. If `maxscl` is 1 then the density and potential file, `STATE.OUT`, will **not** be written to disk at the end of the loop. See `epspt`.

### 5.56 `mixtype`

<code>mixtype</code>	type of mixing required for the potential	integer	1
----------------------	---	---------	---

Currently implemented are:

- 1 Adaptive linear mixing
- 2 Pulay mixing, *Chem. Phys. Lett.* **73**, 393 (1980)
- 3 Broyden mixing, *J. Phys. A: Math. Gen.* **17**, L317 (1984)

### 5.57 mixsdb

mixsdb	subspace dimension for Broyden mixing	integer	5
--------	---------------------------------------	---------	---

This is the number of mixing vectors which define the subspace in which the Hessian matrix is calculated. See `mixtype` and `broydpm`.

### 5.58 mixsdp

mixsdp	subspace dimension for Pulay mixing	integer	3
--------	-------------------------------------	---------	---

See `mixsdb` and `mixtype`.

### 5.59 molecule

molecule	<code>.true.</code> if the system is an isolated molecule	logical	<code>.false.</code>
----------	---	---------	----------------------

If `molecule` is `.true.`, then the atomic positions, `a`, given in the `atoms` block are assumed to be in Cartesian coordinates.

### 5.60 momfix

momfix	the desired total moment for a FSM calculation	real(3)	(0.0,0.0,0.0)
--------	--	---------	---------------

Note that all three components must be specified (even for collinear calculations). See `fixspin`, `taufsm` and `spinpol`.

### 5.61 mommtfix

is	species number	integer	0
ia	atom number	integer	0
mommtfix	the desired muffin-tin moment for a FSM calculation	real(3)	(0.0,0.0,0.0)

The local muffin-tin moments are specified for a subset of atoms, with the list terminated with a blank line. Note that all three components must be specified (even for collinear calculations). See `fixspin`, `taufsm` and `spinpol`.

### 5.62 mstar

mstar	Value of the effective mass parameter used for adaptive determination of <code>swidth</code>	real	10.0
-------	--	------	------

See `autoswidth`.

### 5.63 mustar

mustar	Coulomb pseudopotential, $\mu^*$ , used in the McMillan-Allen-Dynes equation	real	0.15
--------	--	------	------

This is used when calculating the superconducting critical temperature with the formula *Phys. Rev. B* 12, 905 (1975)

$$T_c = \frac{\omega_{\log}}{1.2k_B} \exp \left[ \frac{-1.04(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)} \right],$$

where  $\omega_{\log}$  is the logarithmic average frequency and  $\lambda$  is the electron-phonon coupling constant.

#### 5.64 ncbse

<code>ncbse</code>	number of conduction states to be used for BSE calculations	integer	3
--------------------	---	---------	---

See also `nvbse`.

#### 5.65 ndspem

<code>ndspem</code>	the number of $\mathbf{k}$ -vector displacements in each direction around <code>vklem</code> when computing the numerical derivatives for the effective mass tensor	integer	1
---------------------	---	---------	---

See `deltaem` and `vklem`.

#### 5.66 nempty

<code>nempty</code>	the number of empty states per atom and spin	real	4.0
---------------------	--	------	-----

Defines the number of eigenstates beyond that required for charge neutrality. When running metals it is not known *a priori* how many states will be below the Fermi energy for each  $k$ -point. Setting `nempty` greater than zero allows the additional states to act as a buffer in such cases. Furthermore, magnetic calculations use the first-variational eigenstates as a basis for setting up the second-variational Hamiltonian, and thus `nempty` will determine the size of this basis set. Convergence with respect to this quantity should be checked.

#### 5.67 ngridk

<code>ngridk</code>	the $k$ -point mesh sizes	integer(3)	(1, 1, 1)
---------------------	---------------------------	------------	-----------

The  $\mathbf{k}$ -vectors are generated using

$$\mathbf{k} = \left( \frac{i_1 + v_1}{n_1}, \frac{i_2 + v_2}{n_2}, \frac{i_3 + v_3}{n_3} \right),$$

where  $i_j$  runs from 0 to  $n_j - 1$  and  $0 \leq v_j < 1$  for  $j = 1, 2, 3$ . The vector  $\mathbf{v}$  is given by the variable `vkloff`. See also `reducek`.

#### 5.68 ngridq

<code>ngridq</code>	the phonon $q$ -point mesh sizes	integer(3)	(1, 1, 1)
---------------------	----------------------------------	------------	-----------

Same as `ngridk`, except that this mesh is for the phonon  $q$ -points. See also `reduceq`.

#### 5.69 nosource

<code>nosource</code>	when set to <code>.true.</code> , source fields are projected out of the exchange-correlation magnetic field	logical	<code>.false.</code>
-----------------------	--	---------	----------------------

Experimental feature.

### 5.70 notes

notes(i)	the <i>i</i> th line of the notes	string	-
----------	-----------------------------------	--------	---

This block allows users to add their own notes to the file `INFO.OUT`. The block should be terminated with a blank line, and no line should exceed 80 characters.

### 5.71 nseqit

nseqit	number of iterations per self-consistent loop using the iterative first-variational secular equation solver	integer	30
--------	---	---------	----

See `tseqit`.

### 5.72 ntemp

ntemp	number of temperature steps	integer	20
-------	-----------------------------	---------	----

This is the number of temperature steps to be used in the Eliashberg gap and thermodynamic properties calculations.

### 5.73 nvbse

nvbse	number of valence states to be used for BSE calculations	integer	2
-------	--	---------	---

See also `ncbse`.

### 5.74 nwrite

nwrite	number of self-consistent loops after which <code>STATE.OUT</code> is to be written	integer	0
--------	---	---------	---

Normally, the density and potentials are written to the file `STATE.OUT` only after completion of the self-consistent loop. By setting `nwrite` to a positive integer the file will instead be written every `nwrite` loops.

### 5.75 optcomp

optcomp	the components of the first- or second-order optical tensor to be calculated	integer(3)	(1, 1, 1)
---------	--	------------	-----------

This selects which components of the optical tensor you would like to plot. Only the first two are used for the first-order tensor. Several components can be listed one after the other with a blank line terminating the list.

### 5.76 phwrite

nphwrt	number of <i>q</i> -points for which phonon modes are to be found	integer	1
vqlwrt(i)	the <i>i</i> th <i>q</i> -point in lattice coordinates	real(3)	(0.0, 0.0, 0.0)

This is used in conjunction with `task=230`. The code will write the phonon frequencies and eigenvectors to the file `PHONON.OUT` for all the *q*-points in the list. The *q*-points can

be anywhere in the Brillouin zone and do not have to lie on the mesh defined by `ngridq`. Obviously, all the dynamical matrices have to be computed first using `task=200`.

### 5.77 `plot1d`

<code>nvp1d</code>	number of vertices	integer	2
<code>npp1d</code>	number of plotting points	integer	200
<code>vvlp1d(i)</code>	lattice coordinates for vertex <code>i</code>	real(3)	(0.0, 0.0, 0.0) $\rightarrow$ (1.0, 1.0, 1.0)

Defines the path in either real or reciprocal space along which the 1D plot is to be produced. The user should provide `nvp1d` vertices in lattice coordinates.

### 5.78 `plot2d`

<code>vclp2d(1)</code>	first corner (origin)	real(3)	(0.0, 0.0, 0.0)
<code>vclp2d(2)</code>	second corner	real(3)	(1.0, 0.0, 0.0)
<code>vclp2d(3)</code>	third corner	real(3)	(0.0, 1.0, 0.0)
<code>np2d</code>	number of plotting points in both directions	integer(2)	(40, 40)

Defines the corners of a parallelogram and the grid size used for producing 2D plots.

### 5.79 `plot3d`

<code>vclp3d(1)</code>	first corner (origin)	real(3)	(0.0, 0.0, 0.0)
<code>vclp3d(2)</code>	second corner	real(3)	(1.0, 0.0, 0.0)
<code>vclp3d(3)</code>	third corner	real(3)	(0.0, 1.0, 0.0)
<code>vclp3d(4)</code>	fourth corner	real(3)	(0.0, 0.0, 1.0)
<code>np3d</code>	number of plotting points each direction	integer(3)	(20, 20, 20)

Defines the corners of a box and the grid size used for producing 3D plots.

### 5.80 `primcell`

<code>primcell</code>	<code>.true.</code> if the primitive unit cell should be found	logical	<code>.false.</code>
-----------------------	--	---------	----------------------

Allows the primitive unit cell to be determined automatically from the conventional cell. This is done by searching for lattice vectors among all those which connect atomic sites, and using the three shortest which produce a unit cell with non-zero volume.

### 5.81 `radkpt`

<code>radkpt</code>	radius of sphere used to determine $k$ -point density	real	40.0
---------------------	---	------	------

Used for the automatic determination of the  $k$ -point mesh. If `autokpt` is set to `.true.` then the mesh sizes will be determined by  $n_i = R_k |\mathbf{B}_i| + 1$ , where  $\mathbf{B}_i$  are the primitive reciprocal lattice vectors.

### 5.82 `readalu`

<code>readalu</code>	set to <code>.true.</code> if the interpolation constant for LDA+ $U$ should be read from file rather than calculated	logical	<code>.false.</code>
----------------------	---	---------	----------------------

When `ldapu=3`, the LDA+ $U$  energy and potential are interpolated between FLL and AFM. The interpolation constant,  $\alpha$ , is normally calculated from the density matrix, but can also be read in from the file `ALPHALU.OUT`. This allows the user to fix  $\alpha$ , but is also necessary when calculating forces, since the contribution of the potential of the variation of  $\alpha$  with respect to the density matrix is not computed. See `lda+u`.

### 5.83 `reducebf`

<code>reducebf</code>	reduction factor for the external magnetic fields	real	1.0
-----------------------	---	------	-----

After each self-consistent loop, the external magnetic fields are multiplied with `reducebf`. This allows for a large external magnetic field at the start of the self-consistent loop to break spin symmetry, while at the end of the loop the field will be effectively zero, i.e. infinitesimal. See `bfieldc` and `atoms`.

### 5.84 `reduceh`

<code>reduceh</code>	set to <code>.true.</code> if the reciprocal $\mathbf{H}$ -vectors should be reduced by the symmorphic crystal symmetries	logical	<code>.true.</code>
----------------------	---	---------	---------------------

See `hmaxvr` and `vmat`.

### 5.85 `reducek`

<code>reducek</code>	type of reduction of the $k$ -point set	integer	1
----------------------	---	---------	---

Types of reduction are defined by the symmetry group used:

- 0 no reduction
- 1 reduce with full crystal symmetry group (including non-symmorphic symmetries)
- 2 reduce with symmorphic symmetries only

See also `ngridk` and `vkloff`.

### 5.86 `reduceq`

<code>reduceq</code>	type of reduction of the $q$ -point set	integer	1
----------------------	---	---------	---

See `reducek` and `ngridq`.

### 5.87 `rgkmax`

<code>rgkmax</code>	$R_{\min}^{\text{MT}} \times \max\{ \mathbf{G} + \mathbf{k} \}$	real	7.0
---------------------	---	------	-----

This sets the maximum length for the  $\mathbf{G} + \mathbf{k}$  vectors, defined as `rgkmax` divided by the average muffin-tin radius. See `isgkmax`.

### 5.88 `scale`

<code>scale</code>	lattice vector scaling factor	real	1.0
--------------------	-------------------------------	------	-----

Scaling factor for all three lattice vectors. Applied in conjunction with `scale1`, `scale2` and `scale3`.

### 5.89 `scale1/2/3`

<code>scale1/2/3</code>	separate scaling factors for each lattice vector	real	1.0
-------------------------	--	------	-----

### 5.90 `scissor`

<code>scissor</code>	the scissor correction	real	0.0
----------------------	------------------------	------	-----

This is the scissor shift applied to states above the Fermi energy *Phys. Rev. B* **43**, 4187 (1991). Affects optics calculations only.

### 5.91 `scrpath`

<code>scrpath</code>	scratch space path	string	null
----------------------	--------------------	--------	------

This is the scratch space path where the eigenvector files `EVAlFV.OUT` and `EVAlSV.OUT` will be written. If the run directory is accessed via a network then `scrpath` can be set to a directory on the local disk, for example `/tmp/`. Note that the forward slash `/` at the end of the path must be included.

### 5.92 `socscf`

<code>socscf</code>	scaling factor for the spin-orbit coupling term in the Hamiltonian	real	1.0
---------------------	--	------	-----

This can be used to enhance the effect of spin-orbit coupling in order to accurately determine the magnetic anisotropy energy (MAE).

### 5.93 `spincore`

<code>spincore</code>	set to <code>.true.</code> if the core should be spin-polarised	logical	<code>.false.</code>
-----------------------	---	---------	----------------------

### 5.94 `spinorb`

<code>spinorb</code>	set to <code>.true.</code> if a spin-orbit coupling is required	logical	<code>.false.</code>
----------------------	---	---------	----------------------

If `spinorb` is `.true.`, then a  $\sigma \cdot \mathbf{L}$  term is added to the second-variational Hamiltonian. See `spinpol`.

### 5.95 `spinpol`

<code>spinpol</code>	set to <code>.true.</code> if a spin-polarised calculation is required	logical	<code>.false.</code>
----------------------	--	---------	----------------------

If `spinpol` is `.true.`, then the spin-polarised Hamiltonian is solved as a second-variational step using two-component spinors in the Kohn-Sham magnetic field. The first variational scalar wavefunctions are used as a basis for setting this Hamiltonian.

### 5.96 `spinsprl`

<code>spinsprl</code>	set to <code>.true.</code> if a spin-spiral calculation is required	logical	<code>.false.</code>
-----------------------	---	---------	----------------------

Experimental feature for the calculation of spin-spiral states. See `vqlss` for details.

### 5.97 `sppath`

<code>sppath</code>	path where the species files can be found	string	null
---------------------	---	--------	------

Note that the forward slash / at the end of the path must be included.

### 5.98 `ssdph`

<code>ssdph</code>	set to <code>.true.</code> if a complex de-phasing factor is to be used in spin-spiral calculations	logical	<code>.true.</code>
--------------------	---	---------	---------------------

If this is `.true.` then spin-spiral wavefunctions in each muffin-tin at position  $\mathbf{r}_\alpha$  are de-phased by the matrix

$$\begin{pmatrix} e^{-i\mathbf{q}\cdot\mathbf{r}_\alpha/2} & 0 \\ 0 & e^{i\mathbf{q}\cdot\mathbf{r}_\alpha/2} \end{pmatrix}.$$

In simple situations, this has the advantage of producing magnon dynamical matrices which are already in diagonal form. This option should be used with care, and a full understanding of the spin-spiral configuration is required. See `spinspr1`.

### 5.99 `stype`

<code>stype</code>	integer defining the type of smearing to be used	integer	3
--------------------	--	---------	---

A smooth approximation to the Dirac delta function is needed to compute the occupancies of the Kohn-Sham states. The variable `swidth` determines the width of the approximate delta function. Currently implemented are

- 0 Gaussian
- 1 Methfessel-Paxton order 1, Phys. Rev. B **40**, 3616 (1989)
- 2 Methfessel-Paxton order 2
- 3 Fermi-Dirac

See also `autoswidth` and `swidth`.

### 5.100 `swidth`

<code>swidth</code>	width of the smooth approximation to the Dirac delta function	real	0.001
---------------------	---	------	-------

See `stype` for details.

## 5.101 tasks

task(i)	the <i>i</i> th task	integer	-1
---------	----------------------	---------	----

A list of tasks for the code to perform sequentially. The list should be terminated with a blank line. Each task has an associated integer as follows:

- 1 Write out the version number of the code.
- 0 Ground state run starting from the atomic densities.
- 1 Resumption of ground-state run using density in `STATE.OUT`.
- 2 Geometry optimisation run starting from the atomic densities, with atomic positions written to `GEOMETRY.OUT`.
- 3 Resumption of geometry optimisation run using density in `STATE.OUT` but with positions from `elk.in`.
- 5 Ground state Hartree-Fock run.
- 10 Total, partial and interstitial density of states (DOS).
- 14 Plots the smooth Dirac delta and Heaviside step functions used by the code to calculate occupancies.
- 15 Output **L**, **S** and **J** total expectation values.
- 16 Output **L**, **S** and **J** expectation values for each *k*-point and state in `kstlist`.
- 20 Band structure plot.
- 21 Band structure plot which includes angular momentum characters for every atom.
- 25 Compute the effective mass tensor at the *k*-point given by `vklem`.
- 31, 32, 33 1/2/3D charge density plot.
- 41, 42, 43 1/2/3D exchange-correlation and Coulomb potential plots.
- 51, 52, 53 1/2/3D electron localisation function (ELF) plot.
- 61, 62, 63 1/2/3D wavefunction plot:  $|\Psi_{i\mathbf{k}}(\mathbf{r})|^2$ .
- 65 Write the core wavefunctions to file for plotting.
- 72, 73 2/3D plot of magnetisation vector field,  $\mathbf{m}(\mathbf{r})$ .
- 82, 83 2/3D plot of exchange-correlation magnetic vector field,  $\mathbf{B}_{xc}(\mathbf{r})$ .
- 91, 92, 93 1/2/3D plot of  $\nabla \cdot \mathbf{B}_{xc}(\mathbf{r})$ .
- 100 3D Fermi surface plot using the scalar product  $p(\mathbf{k}) = \Pi_i(\epsilon_{i\mathbf{k}} - \epsilon_F)$ .
- 101 3D Fermi surface plot using separate bands (minus the Fermi energy).
- 102 3D Fermi surface which can be plotted with XCrystDen.
- 105 3D nesting function plot.
- 110 Calculation of Mössbauer contact charge densities and magnetic fields at the nuclear sites.
- 115 Calculation of the electric field gradient (EFG) at the nuclear sites.
- 120 Output of the momentum matrix elements  $\langle \Psi_{i\mathbf{k}} | -i\nabla | \Psi_{j\mathbf{k}} \rangle$ .
- 121 Linear optical dielectric response tensor calculated within the random phase approximation (RPA) and in the  $q \rightarrow 0$  limit, with no microscopic contributions.
- 122 Magneto optical Kerr effect (MOKE) angle.
- 125 Non-linear optical second harmonic generation.
- 130 Output matrix elements of the type  $\langle \Psi_{i\mathbf{k}+\mathbf{q}} | \exp[i(\mathbf{G} + \mathbf{q}) \cdot \mathbf{r}] | \Psi_{j\mathbf{k}} \rangle$ .
- 135 Output all wavefunctions expanded in the plane wave basis up to a cut-off defined by `rgkmax`.

- 140 Energy loss near edge structure (ELNES).  
 142, 143 2/3D plot of the electric field  $\mathbf{E}(\mathbf{r}) \equiv \nabla V_C(\mathbf{r})$ .  
 152, 153 2/3D plot of  $\mathbf{m}(\mathbf{r}) \times \mathbf{B}_{xc}(\mathbf{r})$ .  
 162 Scanning-tunneling microscopy (STM) image.  
 180 Generate the RPA inverse dielectric function with local contributions and write it to file.  
 185 Write the Bethe-Salpeter equation (BSE) Hamiltonian to file.  
 186 Diagonalise the BSE Hamiltonian and write the eigenvectors and eigenvalues to file.  
 187 Output the BSE dielectric response function.  
 190 Write the atomic geometry to file for plotting with XCrySDen and V\_Sim.  
 195 Calculation of X-ray density structure factors.  
 196 Calculation of magnetic structure factors.  
 200 Calculation of phonon dynamical matrices on a  $q$ -point set defined by `ngridq` using the supercell method.  
 201 Phonon dry run: just produce a set of empty DYN files.  
 205 Calculation of phonon dynamical matrices using density functional perturbation theory (DFPT).  
 210 Phonon density of states.  
 220 Phonon dispersion plot.  
 230 Phonon frequencies and eigenvectors for an arbitrary  $q$ -point.  
 240 Generate the  $\mathbf{q}$ -dependent phonon linewidths and electron-phonon coupling constants and write them to file.  
 245 Phonon linewidths plot.  
 250 Eliashberg function  $\alpha^2 F(\omega)$ , electron-phonon coupling constant  $\lambda$ , and the McMillan-Allen-Dynes critical temperature  $T_c$ .  
 300 Reduced density matrix functional theory (RDMFT) calculation.  
 320 Time-dependent density functional theory (TDDFT) calculation of the dielectric response function including microscopic contributions.  
 330 TDDFT calculation of the spin-polarised response function for arbitrary  $\mathbf{q}$ -vectors.  
 400 Calculation of tensor moments and corresponding LDA+U Hartree-Fock energy contributions.

### 5.102 `tau0atp`

<code>tau0atp</code>	the step size to be used for atomic position optimisation	real	0.25
----------------------	---	------	------

The position of atom  $\alpha$  is updated on step  $m$  of a geometry optimisation run using

$$\mathbf{r}_\alpha^{m+1} = \mathbf{r}_\alpha^m + \tau_\alpha^m (\mathbf{F}_\alpha^m + \mathbf{F}_\alpha^{m-1}),$$

where  $\tau_\alpha$  is set to `tau0atp` for  $m = 0$ , and incremented by the same amount if the atom is moving in the same direction between steps. If the direction changes then  $\tau_\alpha$  is reset to `tau0atp`.

### 5.103 `tau0latv`

<code>tau0latv</code>	the step size to be used for lattice vector optimisation	real	0.01
-----------------------	--	------	------

This parameter is used for lattice vector optimisation in a procedure identical to that for atomic position optimisation. See `tau0atp` and `latvopt`.

#### 5.104 `tauoep`

<code>tauoep</code>	step length initial value and scaling factors for the OEP iterative solver	real(3)	(1.0,0.2,1.5)
---------------------	--	---------	---------------

The optimised effective potential is determined using an iterative method. [Phys. Rev. Lett. 98, 196405 (2007)]. At the first iteration the step length is set to `tauoep(1)`. During subsequent iterations, the step length is scaled by `tauoep(2)` or `tauoep(3)`, when the residual is increasing or decreasing, respectively. See also `maxitoeep`.

#### 5.105 `taufsm`

<code>taufsm</code>	the step size to be used when finding the effective magnetic field in fixed spin moment calculations	real	0.01
---------------------	--	------	------

An effective magnetic field,  $\mathbf{B}_{\text{FSM}}$ , is required for fixing the spin moment to a given value,  $\mathbf{M}_{\text{FSM}}$ . This is found by adding a vector to the field which is proportional to the difference between the moment calculated in the  $i$ th self-consistent loop and the required moment:

$$\mathbf{B}_{\text{FSM}}^{i+1} = \mathbf{B}_{\text{FSM}}^i + \lambda (\mathbf{M}^i - \mathbf{M}_{\text{FSM}}),$$

where  $\lambda$  is proportional to `taufsm`. See also `fixspin`, `momfix` and `spinpol`.

#### 5.106 `tfibs`

<code>tfibs</code>	set to <code>.true.</code> if the IBS correction to the force should be calculated	logical	<code>.true.</code>
--------------------	--	---------	---------------------

Because calculation of the incomplete basis set (IBS) correction to the force is fairly time-consuming, it can be switched off by setting `tfibs` to `.false.` This correction can then be included only when necessary, i.e. when the atoms are close to equilibrium in a geometry optimisation run.

#### 5.107 `tforce`

<code>tforce</code>	set to <code>.true.</code> if the force should be calculated at the end of the self-consistent cycle	logical	<code>.false.</code>
---------------------	--	---------	----------------------

This variable is automatically set to `.true.` when performing geometry optimisation.

#### 5.108 `tmomlu`

<code>tmomlu</code>	set to <code>.true.</code> if the tensor moments and the corresponding decomposition of LDA+U energy should be calculated at every loop of the self-consistent cycle	logical	<code>.false.</code>
---------------------	--	---------	----------------------

This variable is useful to check the convergence of the tensor moments in LDA+U calculations. Alternatively, with `task` equal to 400, one can calculate the tensor moments and corresponding LDA+U energy contributions from a given density matrix and set of Slater parameters at the end of the self-consistent cycle.

### 5.109 trimvg

trimvg	set to <code>.true.</code> if the Fourier components of the Kohn-Sham potential for $ G  > \text{gmaxvr}/2$ are to be set to zero	logical	<code>.false.</code>
--------	---	---------	----------------------

This fixes stability problems which can occur for large `rgkmax`. Should be used only in conjunction with large `gmaxvr`.

### 5.110 tseqit

tseqit	set to <code>.true.</code> if the first-variational secular equation should be solved iteratively	logical	<code>.false.</code>
--------	---	---------	----------------------

See also `nseqit`.

### 5.111 tseqr

tseqr	set to <code>.true.</code> if a real symmetric eigenvalue solver should be used for crystals which have inversion symmetry	logical	<code>.true.</code>
-------	--	---------	---------------------

For crystals with inversion symmetry, the first-variational Hamiltonian and overlap matrices can be made real by using appropriate transformations. In this case, a real symmetric (instead of complex Hermitian) eigenvalue solver can be used. This makes the calculation about three times faster.

### 5.112 tshift

tshift	set to <code>.true.</code> if the crystal can be shifted so that the atom closest to the origin is exactly at the origin	logical	<code>.true.</code>
--------	--	---------	---------------------

### 5.113 vhmatt

vhmatt(1)	matrix row 1	real(3)	(1.0,0.0,0.0)
vhmatt(2)	matrix row 2	real(3)	(0.0,1.0,0.0)
vhmatt(3)	matrix row 3	real(3)	(0.0,0.0,1.0)

This is the transformation matrix  $M$  applied to every vector  $\mathbf{H}$  in the structure factor output files `SFACRHO.OUT` and `SFACMAG.OUT`. It is stored in the usual row-column setting and applied directly as  $\mathbf{H}' = M\mathbf{H}$  to every vector but *only* when writing the output files. See also `hmaxvr` and `reduceh`.

### 5.114 vklem

vklem	the $k$ -point in lattice coordinates at which to compute the effective mass tensors	real(3)	(0.0,0.0,0.0)
-------	--	---------	---------------

See `deltaem` and `ndspem`.

### 5.115 vkloff

vkloff	the $k$ -point offset vector in lattice coordinates	real(3)	(0.0,0.0,0.0)
--------	---	---------	---------------

See `ngridk`.

### 5.116 vqlss

<b>vqlss</b>	the $\mathbf{q}$ -vector of the spin-spiral state in lattice coordinates	real(3)	(0.0, 0.0, 0.0)
--------------	--	---------	-----------------

Spin-spirals arise from spinor states assumed to be of the form

$$\Psi_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r}) = \begin{pmatrix} U_{\mathbf{k}}^{\mathbf{q}\uparrow}(\mathbf{r})e^{i(\mathbf{k}+\mathbf{q}/2)\cdot\mathbf{r}} \\ U_{\mathbf{k}}^{\mathbf{q}\downarrow}(\mathbf{r})e^{i(\mathbf{k}-\mathbf{q}/2)\cdot\mathbf{r}} \end{pmatrix}.$$

These are determined using a second-variational approach, and give rise to a magnetisation density of the form

$$\mathbf{m}^{\mathbf{q}}(\mathbf{r}) = (m_x(\mathbf{r}) \cos(\mathbf{q} \cdot \mathbf{r}), m_y(\mathbf{r}) \sin(\mathbf{q} \cdot \mathbf{r}), m_z(\mathbf{r})),$$

where  $m_x$ ,  $m_y$  and  $m_z$  are lattice periodic. See also `spinsprl`.

### 5.117 wplot

<b>nwplot</b>	number of frequency/energy points in the DOS or optics plot	integer	500
<b>ngrkf</b>	fine $k$ -point grid size used for integrating functions in the Brillouin zone	integer	100
<b>nswplot</b>	level of smoothing applied to DOS/optics output	integer	1
<b>wplot</b>	frequency/energy window for the DOS or optics plot	real(2)	(-0.5, 0.5)

DOS and optics plots require integrals of the kind

$$g(\omega_i) = \frac{\Omega}{(2\pi)^3} \int_{\text{BZ}} f(\mathbf{k})\delta(\omega_i - e(\mathbf{k}))d\mathbf{k}.$$

These are calculated by first interpolating the functions  $e(\mathbf{k})$  and  $f(\mathbf{k})$  with the trilinear method on a much finer mesh whose size is determined by `ngrkf`. Then the  $\omega$ -dependent histogram of the integrand is accumulated over the fine mesh. If the output function is noisy then either `ngrkf` should be increased or `nwplot` decreased. Alternatively, the output function can be artificially smoothed up to a level given by `nswplot`. This is the number of successive 3-point averages to be applied to the function  $g$ .

### 5.118 wsfac

<b>wsfac</b>	energy window to be used when calculating density or magnetic structure factors	real(2)	(-10 <sup>6</sup> , 10 <sup>6</sup> )
--------------	---	---------	---------------------------------------

Only those states with eigenvalues within this window will contribute to the density or magnetisation. See also `hmaxvr` and `vhmat`.

### 5.119 xtype

<b>xctype</b>	integers defining the type of exchange-correlation functional to be used	integer(3)	(3, 0, 0)
---------------	--	------------	-----------

Normally only the first value is used to define the functional type. The other value may be used for external libraries. Currently implemented are:

- n* Exact-exchange optimised effective potential (EXX-OEP) method with correlation energy and potential given by functional number *n*
- 1 No exchange-correlation functional ( $E_{xc} \equiv 0$ )
- 2 LDA, Perdew-Zunger/Ceperley-Alder, *Phys. Rev. B* **23**, 5048 (1981)
- 3 LSDA, Perdew-Wang/Ceperley-Alder, *Phys. Rev. B* **45**, 13244 (1992)
- 4 LDA, X-alpha approximation, J. C. Slater, *Phys. Rev.* **81**, 385 (1951)
- 5 LSDA, von Barth-Hedin, *J. Phys. C* **5**, 1629 (1972)
- 20 GGA, Perdew-Burke-Ernzerhof, *Phys. Rev. Lett.* **77**, 3865 (1996)
- 21 GGA, Revised PBE, Zhang-Yang, *Phys. Rev. Lett.* **80**, 890 (1998)
- 22 GGA, PBEsol, *Phys. Rev. Lett.* **100**, 136406 (2008)
- 26 GGA, Wu-Cohen exchange (WC06) with PBE correlation, *Phys. Rev. B* **73**, 235116 (2006)
- 30 GGA, Armiento-Mattsson (AM05) spin-unpolarised functional, *Phys. Rev. B* **72**, 085108 (2005)
- 100 `libxc` functionals, the second and third values of `xctype` define the exchange and correlation functionals in the `libxc` library, respectively

## 6 Contributing to Elk

Please bear in mind when writing code for the Elk project that it should be an exercise in physics and not software engineering. All code should therefore be kept as simple and concise as possible, and above all it should be easy for anyone to locate and follow the Fortran representation of the original mathematics. We would also appreciate the following conventions being adhered to:

- Strict Fortran 90/95 should be used. Features which are marked as obsolescent in F90/95 should be avoided. These include assigned format specifiers, labeled do-loops, computed goto statements and statement functions.
- Modules should be used in place of common blocks for declaring global variables. Use the existing modules to declare new global variables.
- Any code should be written in lower-case free form style, starting from column one. Try and keep the length of each line to fewer than 80 characters using the `&` character for line continuation.
- Every function or subroutine, no matter how small, should be in its own file named `routine.f90`, where `routine` is the function or subroutine name. It is recommended that the routines are named so as to make their purpose apparent from the name alone.
- Use of `implicit none` is mandatory. Remember also to define the `intent` of any passed arguments.
- Local allocatable arrays must be deallocated on exit of the routine to prevent memory leakage. Use of automatic arrays should be limited to arrays of small size.
- Every function or subroutine must be documented with the Protex source code documentation system. This should include a short L<sup>A</sup>T<sub>E</sub>X description of the algorithms and methods involved. Equations which need to be referenced should be labeled with

`routine_1`, `routine_2`, etc. The authorship of each new piece of code or modification should be indicated in the `REVISION HISTORY` part of the header. See the Protex documentation for details.

- Ensure as much as possible that a routine will terminate the program when given improper input instead of continuing with erroneous results. Specifically, functions should have a well-defined domain for which they return accurate results. Input outside that domain should result in an error message and termination.
- Report errors prior to termination with a short description, for example:

```
write(*,*)
write(*,('Error(readinput): natoms <= 0 : ",I8)') natoms(is)
write(*,(' for species ",I4)') is
write(*,*)
stop
```

- Wherever possible, real numbers outputted as ASCII data should be formatted with the `G18.10` specifier.
- Avoid redundant or repeated code: check to see if the routine you need already exists, before writing a new one.
- All reading in of ASCII data should be done in the subroutine `readinput`. For binary data, separate routines for reading and writing should be used (for example `writestate` and `readstate`).
- Input filenames should be in lowercase and have the extension `.in`. All output filenames should be in uppercase with the extension `.OUT`.
- All internal units should be atomic. Input and output units should be atomic by default and clearly stated otherwise. Rydbergs should not be used under any circumstances.

## 6.1 Licensing

Routines which constitute the main part of the code are released under the GNU General Public License (GPL). Library routines are released under the less restrictive GNU Lesser General Public License (LGPL). Both licenses are contained in the file `COPYING`. Any contribution to the code must be licensed at the authors' discretion under either the GPL or LGPL. Author(s) of the code retain the copyrights. Copyright and (L)GPL information must be included at the beginning of every file, and no code will be accepted without this.

## 7 Routine/Function Prologues

### 7.1 allatoms (Source File: allatoms.f90)

INTERFACE:

```
subroutine allatoms
```

*USES:*

```
use modmain
```

DESCRIPTION:

Solves the Kohn-Sham-Dirac equations for each atom type in the solid and finds the self-consistent radial wavefunctions, eigenvalues, charge densities and potentials. The atomic densities can then be used to initialise the crystal densities, and the atomic self-consistent potentials can be appended to the muffin-tin potentials to solve for the core states. Note that, irrespective of the value of `xctype`, exchange-correlation functional type 3 is used. See also `atoms`, `rhoinit`, `gencore` and `modxcifc`.

REVISION HISTORY:

```
Created September 2002 (JKD)
Modified for GGA, June 2007 (JKD)
```

---

### 7.2 atom (Source File: atom.f90)

INTERFACE:

```
subroutine atom(sol,ptnucl,zn,nst,n,l,k,occ,xctype,xcgrad,nr,r,eval,rho,vr,rwf)
```

*USES:*

```
use modxcifc
```

*INPUT/OUTPUT PARAMETERS:*

```
sol      : speed of light in atomic units (in,real)
ptnucl  : .true. if the nucleus is a point particle (in,logical)
zn       : nuclear charge (in,real)
nst      : number of states to solve for (in,integer)
n        : principle quantum number of each state (in,integer(nst))
l        : quantum number l of each state (in,integer(nst))
k        : quantum number k (l or l+1) of each state (in,integer(nst))
occ      : occupancy of each state (inout,real(nst))
xctype   : exchange-correlation type (in,integer(3))
xcgrad   : 1 for GGA functional, 0 otherwise (in,integer)
nr       : number of radial mesh points (in,integer)
```

```

r      : radial mesh (in,real(nr))
eval   : eigenvalue without rest-mass energy for each state (out,real(nst))
rho    : charge density (out,real(nr))
vr     : self-consistent potential (out,real(nr))
rwf    : major and minor components of radial wavefunctions for each state
        (out,real(nr,2,nst))

```

DESCRIPTION:

Solves the Dirac-Kohn-Sham equations for an atom using the exchange-correlation functional `xctype` and returns the self-consistent radial wavefunctions, eigenvalues, charge densities and potentials. Requires the exchange-correlation interface routine `xcifc`.

REVISION HISTORY:

```

Created September 2002 (JKD)
Fixed s.c. convergence problem, October 2003 (JKD)
Added support for GGA functionals, June 2006 (JKD)

```

### 7.3 atpstep (Source File: atpstep.f90)

INTERFACE:

```
subroutine atpstep
```

*USES:*

```
use modmain
use modmpi
```

DESCRIPTION:

Makes a geometry optimisation step and updates the current atomic positions according to the force on each atom. If  $\mathbf{r}_{ij}^m$  is the position and  $\mathbf{F}_{ij}^m$  is the force acting on it for atom  $j$  of species  $i$  and after time step  $m$ , then the new position is calculated by

$$\mathbf{r}_{ij}^{m+1} = \mathbf{r}_{ij}^m + \tau_{ij}^m \left( \mathbf{F}_{ij}^m + \mathbf{F}_{ij}^{m-1} \right),$$

where  $\tau_{ij}^m$  is a parameter governing the size of the displacement. If  $\mathbf{F}_{ij}^m \cdot \mathbf{F}_{ij}^{m-1} > 0$  then  $\tau_{ij}^m$  is increased, otherwise it is decreased.

REVISION HISTORY:

```
Created June 2003 (JKD)
```

## 7.4 axangsu2 (Source File: axangsu2.f90)

subroutine axangsu2(v,th,su2) *INPUT/OUTPUT PARAMETERS:*

v : rotation axis vector (in,real(3))  
th : rotation angle (in,real)  
su2 : SU(2) representation of rotation (out,complex(2,2))

DESCRIPTION:

Finds the complex SU(2) representation of a rotation defined by an axis vector  $\hat{\mathbf{v}}$  and angle  $\theta$ . The spinor rotation matrix is given explicitly by

$$R^{1/2}(\hat{\mathbf{v}}, \theta) = I \cos \frac{\theta}{2} + i(\hat{\mathbf{v}} \cdot \vec{\sigma}) \sin \frac{\theta}{2}.$$

REVISION HISTORY:

Created August 2007 (JKD)  
Reversed rotation direction, February 2008 (L. Nordstrom)

---

## 7.5 bandstr (Source File: bandstr.f90)

INTERFACE:

subroutine bandstr

*USES:*

use modmain

DESCRIPTION:

Produces a band structure along the path in reciprocal space which connects the vertices in the array `vvlp1d`. The band structure is obtained from the second-variational eigenvalues and is written to the file `BAND.OUT` with the Fermi energy set to zero. If required, band structures are plotted to files `BAND_Sss_Aaaaa.OUT` for atom `aaaa` of species `ss`, which include the band characters for each  $l$  component of that atom in columns 4 onwards. Column 3 contains the sum over  $l$  of the characters. Vertex location lines are written to `BANDLINES.OUT`.

REVISION HISTORY:

Created June 2003 (JKD)

---

## 7.6 brzint (Source File: brzint.f90)

### INTERFACE:

```
subroutine brzint(nsm,ngridk,nsk,ikmap,nw,wint,n,ld,e,f,g)
```

### INPUT/OUTPUT PARAMETERS:

```
nsm      : level of smoothing for output function (in,integer)
ngridk   : k-point grid size (in,integer(3))
nsk      : k-point subdivision grid size (in,integer(3))
ikmap    : map from grid to k-point set
           (in,integer(0:ngridk(1)-1,0:ngridk(2)-1,0:ngridk(3)-1))
nw       : number of energy divisions (in,integer)
wint     : energy interval (in,real(2))
n        : number of functions to integrate (in,integer)
ld       : leading dimension (in,integer)
e        : array of energies as a function of k-points (in,real(ld,*))
f        : array of weights as a function of k-points (in,real(ld,*))
g        : output function (out,real(nw))
```

### DESCRIPTION:

Given energy and weight functions,  $e$  and  $f$ , on the Brillouin zone and a set of equidistant energies  $\omega_i$ , this routine computes the integrals

$$g(\omega_i) = \frac{\Omega}{(2\pi)^3} \int_{\text{BZ}} f(\mathbf{k}) \delta(\omega_i - e(\mathbf{k})) d\mathbf{k},$$

where  $\Omega$  is the unit cell volume. This is done by first interpolating  $e$  and  $f$  on a finer  $k$ -point grid using the trilinear method. Then for each  $e(\mathbf{k})$  on the finer grid the nearest  $\omega_i$  is found and  $f(\mathbf{k})$  is accumulated in  $g(\omega_i)$ . If the output function is noisy then either `nsk` should be increased or `nw` decreased. Alternatively, the output function can be artificially smoothed up to a level given by `nsm`. See routine `fsmooth`.

### REVISION HISTORY:

```
Created October 2003 (JKD)
Improved efficiency May 2007 (Sebastian Lebegue)
```

---

## 7.7 charge (Source File: charge.f90)

### INTERFACE:

```
subroutine charge
```

### USES:

```
use modmain
use modtest
```

## DESCRIPTION:

Computes the muffin-tin, interstitial and total charges by integrating the density.

## REVISION HISTORY:

Created April 2003 (JKD)

---

## 7.8 checkmt (Source File: checkmt.f90)

### INTERFACE:

```
subroutine checkmt
```

### USES:

```
use modmain  
use modmpi  
use modvars
```

### DESCRIPTION:

Checks for muffin-tins which are too close together. If any such muffin-tins are found then the radii of their associated atomic species are adjusted so that the minimum distance between their surfaces is `rmtdelta`.

### REVISION HISTORY:

Created May 2003 (JKD)  
Modified, October 2011 (JKD)

---

## 7.9 clebgor (Source File: clebgor.f90)

### INTERFACE:

```
real(8) function clebgor(j1,j2,j3,m1,m2,m3)
```

### INPUT/OUTPUT PARAMETERS:

j1, j2, j3 : angular momentum quantum numbers (in,integer)  
m1, m2, m3 : magnetic quantum numbers (in,integer)

### DESCRIPTION:

Returns the Clebsch-Gordon coefficients using the Wigner  $3j$ -symbols

$$C(J_1 J_2 J_3 | m_1 m_2 m_3) = (-1)^{J_1 - J_2 + m_3} \sqrt{2J_3 + 1} \begin{pmatrix} J_1 & J_2 & J_3 \\ m_1 & m_2 & -m_3 \end{pmatrix}.$$

Suitable for  $J_i \leq 50$ . See `wigner3j`.

### REVISION HISTORY:

Created September 2003 (JKD)

---

## 7.10 connect (Source File: connect.f90)

### INTERFACE:

```
subroutine connect(cvec,nv,np,vv1,vp1,dv,dp)
```

### INPUT/OUTPUT PARAMETERS:

```
cvec : matrix of (reciprocal) lattice vectors stored column-wise
      (in,real(3,3))
nv   : number of vertices (in,integer)
np   : number of connecting points (in,integer)
vv1  : vertex vectors in lattice coordinates (in,real(3,nv))
vp1  : connecting point vectors in lattice coordinates (out,real(3,np))
dv   : cumulative distance to each vertex (out,real(nv))
dp   : cumulative distance to each connecting point (out,real(np))
```

### DESCRIPTION:

Generates a set of points which interpolate between a given set of vertices. Vertex points are supplied in lattice coordinates in the array `vv1` and converted to Cartesian coordinates with the matrix `cvec`. Interpolating points are stored in the array `vp1`. The cumulative distances to the vertices and points along the path are stored in arrays `dv` and `dp`, respectively.

### REVISION HISTORY:

```
Created June 2003 (JKD)
Improved September 2007 (JKD)
Improved again, July 2010 (T. McQueen and JKD)
```

---

## 7.11 dielectric (Source File: dielectric.f90)

### INTERFACE:

```
subroutine dielectric
```

### USES:

```
use modmain
use modtest
```

### DESCRIPTION:

Computes the dielectric tensor, optical conductivity and plasma frequency. The formulae are taken from *Physica Scripta* **T109**, 170 (2004).

### REVISION HISTORY:

Created November 2005 (SS and JKD)  
Added plasma frequency and intraband contribution (S. Lebegue)  
Complete rewrite, 2008 (JKD)  
Fixed problem with plasma frequency, 2009 (Marty Blaber and JKD)  
Parallelised, 2009 (M. Blaber)

---

## 7.12 dos (Source File: dos.f90)

### INTERFACE:

```
subroutine dos
```

### USES:

```
use modmain  
use modtest
```

### DESCRIPTION:

Produces a total and partial density of states (DOS) for plotting. The total DOS is written to the file TDOS.OUT while the partial DOS is written to the file PDOS\_Sss\_Aaaaa.OUT for atom aaaa of species ss. In the case of the partial DOS, each symmetrised  $(l, m)$ -projection is written consecutively and separated by blank lines. If the global variable `lmirep` is `.true.`, then the density matrix from which the  $(l, m)$ -projections are obtained is first rotated into a irreducible representation basis, i.e. one that block diagonalises all the site symmetry matrices in the  $Y_{lm}$  basis. Eigenvalues of a quasi-random matrix in the  $Y_{lm}$  basis which has been symmetrised with the site symmetries are written to ELMIREP.OUT. This allows for identification of the irreducible representations of the site symmetries, for example  $e_g$  or  $t_{2g}$ , by the degeneracies of the eigenvalues. In the plot, spin-up is made positive and spin-down negative. See the routines `gendmat` and `brzint`.

### REVISION HISTORY:

Created January 2004 (JKD)  
Parallelised and included sum over m, November 2009 (F. Cricchio)

---

## 7.13 elfplot (Source File: elfplot.f90)

### INTERFACE:

```
subroutine elfplot
```

### USES:

```
use modmain
```

## DESCRIPTION:

Outputs the electron localisation function (ELF) for 1D, 2D or 3D plotting. The spin-averaged ELF is given by

$$f_{\text{ELF}}(\mathbf{r}) = \frac{1}{1 + [D(\mathbf{r})/D^0(\mathbf{r})]^2},$$

where

$$D(\mathbf{r}) = \frac{1}{2} \left( \tau(\mathbf{r}) - \frac{1}{4} \frac{[\nabla n(\mathbf{r})]^2}{n(\mathbf{r})} \right)$$

and

$$\tau(\mathbf{r}) = \sum_{i=1}^N |\nabla \Psi_i(\mathbf{r})|^2$$

is the spin-averaged kinetic energy density from the spinor wavefunctions. The function  $D^0$  is the kinetic energy density for the homogeneous electron gas evaluated for  $n(\mathbf{r})$ :

$$D^0(\mathbf{r}) = \frac{3}{5} (6\pi^2)^{2/3} \left( \frac{n(\mathbf{r})}{2} \right)^{5/3}.$$

The ELF is useful for the topological classification of bonding. See for example T. Burnus, M. A. L. Marques and E. K. U. Gross [Phys. Rev. A 71, 10501 (2005)].

## REVISION HISTORY:

Created September 2003 (JKD)

Fixed bug found by F. Wagner (JKD)

---

## 7.14 eliashberg (Source File: eliashberg.f90)

### INTERFACE:

```
subroutine eliashberg
```

### USES:

```
use modmain
use modphonon
```

### DESCRIPTION:

Calculates the superconducting gap within Eliashberg theory. This implementation is isotropic and assumes a flat density of states. The Eliashberg function  $\alpha^2F$  is required as input for this calculation.

### REVISION HISTORY:

Created December 2010 (Antonio Sanna)

Modified, June 2011 (JKD)

---

## 7.15 energy (Source File: energy.f90)

INTERFACE:

```
subroutine energy
```

USES:

```
use modmain
use modldapu
use modtest
```

DESCRIPTION:

Computes the total energy and its individual contributions. The kinetic energy is given by

$$T_s = \sum_i n_i \epsilon_i - \int \rho(\mathbf{r}) [v_C(\mathbf{r}) + v_{xc}(\mathbf{r})] d\mathbf{r} - \int \mathbf{m}(\mathbf{r}) \cdot (\mathbf{B}_{xc}(\mathbf{r}) + \mathbf{B}_{ext}(\mathbf{r})) d\mathbf{r},$$

where  $n_i$  are the occupancies and  $\epsilon_i$  are the eigenvalues of both the core and valence states;  $\rho$  is the density;  $\mathbf{m}$  is the magnetisation density;  $v_C$  is the Coulomb potential;  $v_{xc}$  and  $\mathbf{B}_{xc}$  are the exchange-correlation potential and magnetic field, respectively; and  $\mathbf{B}_{ext}$  is the external magnetic field. The Hartree, electron-nuclear and nuclear-nuclear electrostatic energies are combined into the Coulomb energy:

$$\begin{aligned} E_C &= E_H + E_{en} + E_{nn} \\ &= \frac{1}{2} V_C + E_{Mad}, \end{aligned}$$

where

$$V_C = \int \rho(\mathbf{r}) v_C(\mathbf{r}) d\mathbf{r}$$

is the Coulomb potential energy. The Madelung energy is given by

$$E_{Mad} = \frac{1}{2} \sum_{\alpha} z_{\alpha} R_{\alpha},$$

where

$$R_{\alpha} = \lim_{r \rightarrow 0} \left( v_{\alpha;00}^C(r) Y_{00} + \frac{z_{\alpha}}{r} \right)$$

for atom  $\alpha$ , with  $v_{\alpha;00}^C$  being the  $l = 0$  component of the spherical harmonic expansion of  $v_C$  in the muffin-tin, and  $z_{\alpha}$  is the nuclear charge. Using the nuclear-nuclear energy determined at the start of the calculation, the electron-nuclear and Hartree energies can be isolated with

$$E_{en} = 2(E_{Mad} - E_{nn})$$

and

$$E_H = \frac{1}{2}(E_C - E_{en}).$$

Finally, the total energy is

$$E = T_s + E_C + E_{xc},$$

where  $E_{xc}$  is obtained either by integrating the exchange-correlation energy density, or in the case of exact exchange, the explicit calculation of the Fock exchange integral. The energy from the external magnetic fields in the muffin-tins, `bfcmt`, is always removed from the total since these fields are non-physical: their field lines do not close. The energy of the physical external field, `bfieldc`, is also not included in the total because this field, like those in the muffin-tins, is used for breaking spin symmetry and taken to be infinitesimal. If this field is intended to be finite, then the associated energy, `engybext`, should be added to the total by hand. See `potxc`, `exxengy` and related subroutines.

#### REVISION HISTORY:

Created May 2003 (JKD)

---

### 7.16 `engyflu` (Source File: `engyflu.f90`)

#### INTERFACE:

```
subroutine engyflu
```

#### *USES:*

```
use modmain  
use modldapu
```

#### DESCRIPTION:

Calculates the energies of radial functions to be used to calculate the Slater integrals. By convention those energies are chosen to be the ones at the center of the band.

#### REVISION HISTORY:

Created April 2008 from `linengy` (Francesco Cricchio)

---

### 7.17 `erf` (Source File: `erf.f90`)

#### INTERFACE:

```
real(8) function erf(x)
```

#### *INPUT/OUTPUT PARAMETERS:*

```
x : real argument (in,real)
```

#### DESCRIPTION:

Returns the error function  $\text{erf}(x)$  using a rational function approximation. This procedure is numerically stable and accurate to near machine precision.

#### REVISION HISTORY:

Modified version of a NSW routine, April 2003 (JKD)

---

## 7.18 euler (Source File: euler.f90)

INTERFACE:

```
subroutine euler(rot,ang)
```

*INPUT/OUTPUT PARAMETERS:*

```
rot : rotation matrix (in,real(3,3))  
ang : euler angles (alpha, beta, gamma) (out,real(3))
```

DESCRIPTION:

Given a rotation matrix

$$R(\alpha, \beta, \gamma) = \begin{pmatrix} \cos \gamma \cos \beta \cos \alpha - \sin \gamma \sin \alpha & \cos \gamma \cos \beta \sin \alpha + \sin \gamma \cos \alpha & -\cos \gamma \sin \beta \\ -\sin \gamma \cos \beta \cos \alpha - \cos \gamma \sin \alpha & -\sin \gamma \cos \beta \sin \alpha + \cos \gamma \cos \alpha & \sin \gamma \sin \beta \\ \sin \beta \cos \alpha & \sin \beta \sin \alpha & \cos \beta \end{pmatrix},$$

this routine determines the Euler angles,  $(\alpha, \beta, \gamma)$ . This corresponds to the so-called “y-convention”, which involves the following successive rotations of the coordinate system:

1. The  $x'_1$ -,  $x'_2$ -,  $x'_3$ -axes are rotated anticlockwise through an angle  $\alpha$  about the  $x_3$  axis
2. The  $x''_1$ -,  $x''_2$ -,  $x''_3$ -axes are rotated anticlockwise through an angle  $\beta$  about the  $x'_2$  axis
3. The  $x'''_1$ -,  $x'''_2$ -,  $x'''_3$ -axes are rotated anticlockwise through an angle  $\gamma$  about the  $x''_3$  axis

Note that the Euler angles are not necessarily unique for a given rotation matrix.

REVISION HISTORY:

```
Created May 2003 (JKD)  
Fixed problem thanks to Frank Wagner, June 2013 (JKD)
```

---

## 7.19 factnm (Source File: factnm.f90)

INTERFACE:

```
real(8) function factnm(n,m)
```

*INPUT/OUTPUT PARAMETERS:*

```
n : input (in,integer)  
m : order of multifactorial (in,integer)
```

DESCRIPTION:

Returns the multifactorial

$$n \underbrace{!! \dots!}_{m \text{ times}} = \prod_{\substack{i \geq 0 \\ n - im > 0}} (n - im)$$

for  $n, m \geq 0$ .  $n$  should be less than 150.

REVISION HISTORY:

Created January 2003 (JKD)

---

## 7.20 factr (Source File: factr.f90)

INTERFACE:

```
real(8) function factr(n,d)
```

*INPUT/OUTPUT PARAMETERS:*

```
  n : numerator (in,integer)
  d : denominator (in,integer)
```

DESCRIPTION:

Returns the ratio  $n!/d!$  for  $n, d \geq 0$ . Performs no under- or overflow checking.

REVISION HISTORY:

Created October 2002 (JKD)

---

## 7.21 fderiv (Source File: fderiv.f90)

INTERFACE:

```
subroutine fderiv(m,n,x,f,g)
```

*INPUT/OUTPUT PARAMETERS:*

```
  m : order of derivative (in,integer)
  n : number of points (in,integer)
  x : abscissa array (in,real(n))
  f : function array (in,real(n))
  g : (anti-)derivative of f (out,real(n))
```

## DESCRIPTION:

Given function  $f$  defined on a set of points  $x_i$  then if  $m \geq 0$  this routine computes the  $m$ th derivative of  $f$  at each point. If  $m < 0$  the anti-derivative of  $f$  given by

$$g(x_i) = \int_{x_1}^{x_i} f(x) dx$$

is calculated. If  $m = -1$  then an accurate integral is computed by fitting the function to a clamped cubic spline. When  $m = -3$  the fast but low accuracy trapezoidal integration method is used. Simpson's integration, which is slower but more accurate than the trapezoidal method, is used if  $m = -2$ .

## REVISION HISTORY:

Created May 2002 (JKD)

---

## 7.22 findband (Source File: findband.f90)

### INTERFACE:

```
subroutine findband(sol,l,k,nr,r,vr,eps,demax,e,fnd)
```

### INPUT/OUTPUT PARAMETERS:

```
sol   : speed of light in atomic units (in,real)
l     : angular momentum quantum number (in,integer)
k     : quantum number k, zero if Dirac eqn. is not to be used (in,integer)
nr    : number of radial mesh points (in,integer)
r     : radial mesh (in,real(nr))
vr    : potential on radial mesh (in,real(nr))
eps   : energy search tolerance (in,real)
demax : maximum allowed change from the input energy; enforced only if e < 0
       (in,real)
e     : input energy and returned band energy (inout,real)
fnd   : set to .true. if the band energy is found (out,logical)
```

### DESCRIPTION:

Finds the band energies for a given radial potential and angular momentum. This is done by first searching upwards in energy, starting from the input energy plus the offset energy, until the radial wavefunction at the muffin-tin radius is zero. This is the energy at the top of the band, denoted  $E_t$ . A downward search is now performed from  $E_t$  until the slope of the radial wavefunction at the muffin-tin radius is zero. This energy,  $E_b$ , is at the bottom of the band. The band energy is taken as  $(E_t + E_b)/2$ . If either  $E_t$  or  $E_b$  cannot be found then the band energy is set to the input value.

### REVISION HISTORY:

Created September 2004 (JKD)

Added two-pass loop, October 2013 (JKD)

---

## 7.23 findlambdalU (Source File: findlambdalU.f90)

### INTERFACE:

```
subroutine findlambdalU(is,l,ufix,lambdA0,lambdA)
```

### INPUT/OUTPUT PARAMETERS:

```
is      : species type (in,integer)
l       : angular momentum (in,integer)
ufix    : fixed U (in,integer)
lambdA0 : starting value for screening length (inout,real)
lambdA  : screening length corresponding to fixed U (out,real)
```

### DESCRIPTION:

Find the screening length corresponding to a fixed value of  $U$  by using the half-interval method in the first few steps and then the more efficient secant method. For  $U = 0$  the code automatically sets the screening length to  $\text{lambdamax} = 50$ . This value is enough to get  $F^{(k)} \sim 10^{-3}$  corresponding to  $U \sim 0$  (that perfectly mimics a bare LDA calculation). However the code is stable up to  $\text{lambdamax} \sim 200$  thanks to improvement of `spline`.

### REVISION HISTORY:

Created July 2009 (Francesco Cricchio)

---

## 7.24 findngkmax (Source File: findngkmax.f90)

### INTERFACE:

```
subroutine findngkmax(nkpt,vkc,nspnfv,vqcSS,ngvec,vgc,gkmax,ngkmax)
```

### INPUT/OUTPUT PARAMETERS:

```
nkpt    : number of k-points (in,integer)
vkc     : k-point vectors in Cartesian coordinates (in,real(3,nkpt))
nspnfv  : number of first-variational spin components: 1 normal case, 2 for
          spin-spiral case (in,integer)
vqcSS   : spin-spiral q-vector, not referenced if nspnfv=1 (in,integer)
ngvec   : number of G-vectors (in,integer)
vgc     : G-vectors in Cartesian coordinates (in,real(3,ngvec))
gkmax   : maximum allowed |G+k| (in,real)
ngkmax  : maximum number of G+k-vectors over all k-points (out,integer)
```

### DESCRIPTION:

Determines the largest number of  $\mathbf{G} + \mathbf{k}$ -vectors with length less than  $\text{gkmax}$  over all the  $k$ -points. This variable is used for allocating arrays.

### REVISION HISTORY:

Created October 2004 (JKD)  
Modified, August 2012 (JKD)  
Removed modmain and added arguments, September 2012 (JKD)

---

## 7.25 findprimcell (Source File: findprimcell.f90)

INTERFACE:

```
subroutine findprimcell
```

USES:

```
use modmain
```

DESCRIPTION:

This routine finds the smallest primitive cell which produces the same crystal structure as the conventional cell. This is done by searching through all the vectors which connect atomic positions and finding those which leave the crystal structure invariant. Of these, the three shortest which produce a non-zero unit cell volume are chosen.

REVISION HISTORY:

Created April 2007 (JKD)

---

## 7.26 findswidth (Source File: findswidth.f90)

INTERFACE:

```
subroutine findswidth
```

USES:

```
use modmain
```

DESCRIPTION:

Calculates the smearing width from the  $k$ -point density,  $V_{\text{BZ}}/n_k$ ; the valence band width,  $W$ ; and an effective mass parameter,  $m^*$ ; according to

$$\sigma = \frac{\sqrt{2W}}{m^*} \left( \frac{3}{4\pi} \frac{V_{\text{BZ}}}{n_k} \right)^{1/3}.$$

The valence bandwidth is determined by stepping down in energy from the Fermi level until a gap larger than a given tolerance is found. This method was presented in T. Björkman and O. Grånäs, *Int. J. Quant. Chem.* DOI: 10.1002/qua.22476.

REVISION HISTORY:

Created April 2010 (Torbjorn Bjorkman and JKD)

---

## 7.27 findsymcrys (Source File: findsymcrys.f90)

### INTERFACE:

```
subroutine findsymcrys
```

### USES:

```
use modmain
use modmpi
use modtest
```

### DESCRIPTION:

Finds the complete set of symmetries which leave the crystal structure (including the magnetic fields) invariant. A crystal symmetry is of the form  $\{\alpha_S|\alpha_R|\mathbf{t}\}$ , where  $\mathbf{t}$  is a translation vector,  $\alpha_R$  is a spatial rotation operation and  $\alpha_S$  is a global spin rotation. Note that the order of operations is important and defined to be from right to left, i.e. translation followed by spatial rotation followed by spin rotation. In the case of spin-orbit coupling  $\alpha_S = \alpha_R$ . In order to determine the translation vectors, the entire atomic basis is shifted so that the first atom in the smallest set of atoms of the same species is at the origin. Then all displacement vectors between atoms in this set are checked as possible symmetry translations. If the global variable `tshift` is set to `.false.` then the shift is not performed. See L. M. Sandratskii and P. G. Guletskii, *J. Phys. F: Met. Phys.* **16**, L43 (1986) and the routine `findsym`.

### REVISION HISTORY:

Created April 2007 (JKD)

---

## 7.28 findsym (Source File: findsym.f90)

### INTERFACE:

```
subroutine findsym(apl1,apl2,nsym,lspl,lspn,iea)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
apl1 : first set of atomic positions in lattice coordinates
      (in,real(3,maxatoms,maxspecies))
apl2 : second set of atomic positions in lattice coordinates
      (in,real(3,maxatoms,maxspecies))
nsym : number of symmetries (out,integer)
lspl : spatial rotation element in lattice point group for each symmetry
      (out,integer(48))
```

```

lspn : spin rotation element in lattice point group for each symmetry
      (out,integer(48))
iea  : equivalent atom index for each symmetry
      (out,integer(iea(natmmax,nspecies,48)))

```

DESCRIPTION:

Finds the symmetries which rotate one set of atomic positions into another. Both sets of positions differ only by a translation vector and have the same muffin-tin magnetic fields (stored in the global array `bfcmt`). Any symmetry element consists of a spatial rotation of the atomic position vectors followed by a global magnetic rotation:  $\{\alpha_S|\alpha_R\}$ . In the case of spin-orbit coupling  $\alpha_S = \alpha_R$ . The symmetries are returned as indices of elements in the Bravais lattice point group. An index to equivalent atoms is stored in the array `iea`.

REVISION HISTORY:

```

Created April 2007 (JKD)
Fixed use of proper rotations for spin, February 2008 (L. Nordstrom)

```

---

## 7.29 findsymlat (Source File: findsymlat.f90)

INTERFACE:

```
subroutine findsymlat
```

USES:

```
use modmain
```

DESCRIPTION:

Finds the point group symmetries which leave the Bravais lattice invariant. Let  $A$  be the matrix consisting of the lattice vectors in columns, then

$$g = A^T A$$

is the metric tensor. Any  $3 \times 3$  matrix  $S$  with elements  $-1, 0$  or  $1$  is a point group symmetry of the lattice if  $\det(S)$  is  $-1$  or  $1$ , and

$$S^T g S = g.$$

The first matrix in the set returned is the identity.

REVISION HISTORY:

```

Created January 2003 (JKD)
Removed arguments and simplified, April 2007 (JKD)

```

---

### 7.30 flushfc (Source File: flushfc.f90)

#### INTERFACE:

```
subroutine flushfc(fnum)
```

#### INPUT/OUTPUT PARAMETERS:

```
fnum : unit specifier for file (in,integer)
```

#### DESCRIPTION:

Interface to the Fortran `flush` statement. Some compilers do not support the `flush` command, which is very useful for keeping small formatted files up-to-date on the disk. The routine implimented below is a machine-independent emulation of `flush`, but may be replaced with the intrinsic command if preferred.

#### REVISION HISTORY:

```
Created September 2002 (JKD)
```

---

### 7.31 fluykwa0 (Source File: fluykwa0.f90)

#### INTERFACE:

```
real(8) function fluykwa0(is,l1,k)
```

#### USES:

```
use modmain  
use modldapu
```

#### INPUT/OUTPUT PARAMETERS:

```
is : species type (in,integer)  
l  : an angular momentum (in,integer)  
k  : order of Slater parameter (in,integer)
```

#### DESCRIPTION:

Calculates the Slater parameters in the unscreened case. See *Phys. Rev. B* **52**, 1421 (1995) and *Phys. Rev. B* **80**, 035121 (2009).

#### REVISION HISTORY:

```
Created April 2008 (LN)  
Modified and tested July 2008 (LN and FC)
```

---

### 7.32 fluykwa (Source File: fluykwa.f90)

INTERFACE:

```
real(8) function fluykwa(is,l,k,lambda)
```

USES:

```
use modmain
use modldapu
```

INPUT/OUTPUT PARAMETERS:

```
is      : species type (in,integer)
l       : an angular momentum (in,integer)
k       : order of Slater parameter (in,integer)
lambda : screening length of Yukawa potential (in,real)
```

DESCRIPTION:

Calculates the Slater parameters using a screened Yukawa potential. See *Phys. Rev. B* **52**, 1421 (1995) and *Phys. Rev. B* **80**, 035121 (2009).

REVISION HISTORY:

```
Created April 2008 (Lars Nordstrom)
Modified and tested July 2008 (LN and FC)
```

---

### 7.33 force (Source File: force.f90)

INTERFACE:

```
subroutine force
```

USES:

```
use modmain
use modtest
use modmpi
```

DESCRIPTION:

Computes the various contributions to the atomic forces. In principle, the force acting on a nucleus is simply the gradient at that site of the classical electrostatic potential from the other nuclei and the electronic density. This is a result of the Hellmann-Feynman theorem. However because the basis set is dependent on the nuclear coordinates and is not complete, the Hellman-Feynman force is inaccurate and corrections to it are required. The first is the core correction which arises because the core wavefunctions were determined by neglecting the non-spherical parts of the Kohn-Sham potential  $v_s$ . Explicitly this is given by

$$\mathbf{F}_{\text{core}}^{\alpha} = \int_{\text{MT}_{\alpha}} v_s(\mathbf{r}) \nabla \rho_{\text{core}}^{\alpha}(\mathbf{r}) d\mathbf{r}$$

for atom  $\alpha$ . The second, which is the incomplete basis set (IBS) correction, is due to the position dependence of the APW functions, and is derived by considering the change in total energy if the eigenvector coefficients were fixed and the APW functions themselves were changed. This would result in changes to the first-variational Hamiltonian and overlap matrices given by

$$\begin{aligned}\delta H_{\mathbf{G},\mathbf{G}'}^\alpha &= i(\mathbf{G} - \mathbf{G}') \left( H_{\mathbf{G}+\mathbf{k},\mathbf{G}'+\mathbf{k}}^\alpha - \frac{1}{2}(\mathbf{G} + \mathbf{k}) \cdot (\mathbf{G}' + \mathbf{k}) \tilde{\Theta}_\alpha(\mathbf{G} - \mathbf{G}') e^{-i(\mathbf{G}-\mathbf{G}')\cdot\mathbf{r}_\alpha} \right) \\ \delta O_{\mathbf{G},\mathbf{G}'}^\alpha &= i(\mathbf{G} - \mathbf{G}') \left( O_{\mathbf{G}+\mathbf{k},\mathbf{G}'+\mathbf{k}}^\alpha - \tilde{\Theta}_\alpha(\mathbf{G} - \mathbf{G}') e^{-i(\mathbf{G}-\mathbf{G}')\cdot\mathbf{r}_\alpha} \right)\end{aligned}$$

where both  $\mathbf{G}$  and  $\mathbf{G}'$  run over the APW indices;  $\tilde{\Theta}_\alpha$  is the form factor of the smooth step function for muffin-tin  $\alpha$ ; and  $H^\alpha$  and  $O^\alpha$  are the muffin-tin Hamiltonian and overlap matrices, respectively. The APW-local-orbital part is given by

$$\begin{aligned}\delta H_{\mathbf{G},\mathbf{G}'}^\alpha &= i(\mathbf{G} + \mathbf{k}) H_{\mathbf{G}+\mathbf{k},\mathbf{G}'+\mathbf{k}}^\alpha \\ \delta O_{\mathbf{G},\mathbf{G}'}^\alpha &= i(\mathbf{G} + \mathbf{k}) O_{\mathbf{G}+\mathbf{k},\mathbf{G}'+\mathbf{k}}^\alpha\end{aligned}$$

where  $\mathbf{G}$  runs over the APW indices and  $\mathbf{G}'$  runs over the local-orbital indices. There is no contribution from the local-orbital-local-orbital part of the matrices. We can now write the IBS correction in terms of the basis of first-variational states as

$$\mathbf{F}_{ij}^{\alpha\mathbf{k}} = \sum_{\mathbf{G},\mathbf{G}'} b_{\mathbf{G}}^{i\mathbf{k}*} b_{\mathbf{G}'}^{j\mathbf{k}} (\delta H_{\mathbf{G},\mathbf{G}'}^\alpha - \epsilon_j \delta O_{\mathbf{G},\mathbf{G}'}^\alpha),$$

where  $b^{i\mathbf{k}}$  is the first-variational eigenvector. Finally, the  $\mathbf{F}_{ij}^{\alpha\mathbf{k}}$  matrix elements can be multiplied by the second-variational coefficients, and contracted over all indices to obtain the IBS force:

$$\mathbf{F}_{\text{IBS}}^\alpha = \sum_{\mathbf{k}} w_{\mathbf{k}} \sum_{l\sigma} n_{l\mathbf{k}} \sum_{ij} c_{\sigma i}^{l\mathbf{k}*} c_{\sigma j}^{l\mathbf{k}} \mathbf{F}_{ij}^{\alpha\mathbf{k}} + \int_{\text{MT}_\alpha} v_s(\mathbf{r}) \nabla [\rho(\mathbf{r}) - \rho_{\text{core}}^\alpha(\mathbf{r})] d\mathbf{r},$$

where  $c^{l\mathbf{k}}$  are the second-variational coefficients,  $w_{\mathbf{k}}$  are the  $k$ -point weights,  $n_{l\mathbf{k}}$  are the occupancies. See routines `hmlaa`, `olpaa`, `hmlalo`, `olpalo`, `energy`, `seceqn` and `gencfun`.

REVISION HISTORY:

Created January 2004 (JKD)

Fixed problem with second-variational forces, May 2008 (JKD)

### 7.34 forcek (Source File: forcek.f90)

INTERFACE:

```
subroutine forcek(ik)
```

USES:

```
use modmain
```

## DESCRIPTION:

Computes the  $\mathbf{k}$ -dependent contribution to the incomplete basis set (IBS) force. See the calling routine `force` for a full description.

## REVISION HISTORY:

Created June 2006 (JKD)

Updated for spin-spiral case, May 2007 (Francesco Cricchio and JKD)

---

### 7.35 fsmfield (Source File: fsmfield.f90)

#### INTERFACE:

```
subroutine fsmfield
```

#### USES:

```
use modmain
```

#### DESCRIPTION:

Updates the effective magnetic field,  $\mathbf{B}_{\text{FSM}}$ , required for fixing the spin moment to a given value,  $\mathbf{M}_{\text{FSM}}$ . This is done by adding a vector to the field which is proportional to the difference between the moment calculated in the  $i$ th self-consistent loop and the required moment:

$$\mathbf{B}_{\text{FSM}}^{i+1} = \mathbf{B}_{\text{FSM}}^i + \lambda (\mathbf{M}^i - \mathbf{M}_{\text{FSM}}),$$

where  $\lambda$  is a scaling factor.

#### REVISION HISTORY:

Created March 2005 (JKD)

---

### 7.36 fsmooth (Source File: fsmooth.f90)

#### INTERFACE:

```
subroutine fsmooth(m,n,ld,f)
```

#### INPUT/OUTPUT PARAMETERS:

`m` : number of 3-point running averages to perform (in,integer)  
`n` : number of point (in,integer)  
`ld` : leading dimension (in,integer)  
`f` : function array (inout,real(ld,n))

## DESCRIPTION:

Removes numerical noise from a function by performing  $m$  successive 3-point running averages on the data. The endpoints are kept fixed.

## REVISION HISTORY:

Created December 2005 (JKD)

---

### 7.37 gaunt (Source File: gaunt.f90)

#### INTERFACE:

```
real(8) function gaunt(l1,l2,l3,m1,m2,m3)
```

#### INPUT/OUTPUT PARAMETERS:

l1, l2, l3 : angular momentum quantum numbers (in,integer)

m1, m2, m3 : magnetic quantum numbers (in,integer)

#### DESCRIPTION:

Returns the Gaunt coefficient given by

$$\langle Y_{m_1}^{l_1} | Y_{m_2}^{l_2} | Y_{m_3}^{l_3} \rangle = (-1)^{m_1} \left[ \frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi} \right]^{\frac{1}{2}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l_3 \\ -m_1 & m_2 & m_3 \end{pmatrix}.$$

Suitable for  $l_i$  less than 50.

#### REVISION HISTORY:

Created November 2002 (JKD)

---

### 7.38 gauntyry (Source File: gauntyry.f90)

#### INTERFACE:

```
complex(8) function gauntyry(l1,l2,l3,m1,m2,m3)
```

#### INPUT/OUTPUT PARAMETERS:

l1, l2, l3 : angular momentum quantum numbers (in,integer)

m1, m2, m3 : magnetic quantum numbers (in,integer)

#### DESCRIPTION:

Returns the complex Gaunt-like coefficient given by  $\langle Y_{m_1}^{l_1} | R_{m_2}^{l_2} | Y_{m_3}^{l_3} \rangle$ , where  $Y_{lm}$  and  $R_{lm}$  are the complex and real spherical harmonics, respectively. Suitable for  $l_i$  less than 50. See routine `genrlm`.

#### REVISION HISTORY:

Created November 2002 (JKD)

---

### 7.39 gcd (Source File: gcd.f90)

INTERFACE:

```
integer function gcd(x,y)
```

*INPUT/OUTPUT PARAMETERS:*

```
  x : first integer (in,integer)
  y : second integer (in,integer)
```

DESCRIPTION:

Computes the greatest common divisor (GCD) of two integers using Euclid's algorithm.

REVISION HISTORY:

Created September 2004 (JKD)

---

### 7.40 genapwfr (Source File: genapwfr.f90)

INTERFACE:

```
subroutine genapwfr
```

*USES:*

```
use modmain
```

DESCRIPTION:

Generates the APW radial functions. This is done by integrating the scalar relativistic Schrödinger equation (or its energy derivatives) at the current linearisation energies using the spherical part of the Kohn-Sham potential. The number of radial functions at each  $l$ -value is given by the variable `apword` (at the muffin-tin boundary, the APW functions have continuous derivatives up to order `apword - 1`). Within each  $l$ , these functions are orthonormalised with the Gram-Schmidt method. The radial Hamiltonian is applied to the orthonormalised functions and the results are stored in the global array `apwfr`.

REVISION HISTORY:

Created March 2003 (JKD)  
Copied to equivalent atoms, February 2010 (A. Kozhevnikov and JKD)

---

## 7.41 gencfun (Source File: gencfun.f90)

INTERFACE:

```
subroutine gencfun
```

USES:

```
use modmain
```

DESCRIPTION:

Generates the smooth characteristic function. This is the function which is 0 within the muffin-tins and 1 in the interstitial region and is constructed from radial step function form-factors with  $G < G_{\max}$ . The form factors are given by

$$\tilde{\Theta}_i(G) = \begin{cases} \frac{4\pi R_i^3}{3\Omega} & G = 0 \\ \frac{4\pi R_i^3}{\Omega} \frac{j_1(GR_i)}{GR_i} & 0 < G \leq G_{\max} \\ 0 & G > G_{\max} \end{cases},$$

where  $R_i$  is the muffin-tin radius of the  $i$ th species and  $\Omega$  is the unit cell volume. Therefore the characteristic function in  $G$ -space is

$$\tilde{\Theta}(\mathbf{G}) = \delta_{G,0} - \sum_{ij} \exp(-i\mathbf{G} \cdot \mathbf{r}_{ij}) \tilde{\Theta}_i(G),$$

where  $\mathbf{r}_{ij}$  is the position of the  $j$ th atom of the  $i$ th species.

REVISION HISTORY:

Created January 2003 (JKD)

---

## 7.42 gencore (Source File: gencore.f90)

INTERFACE:

```
subroutine gencore
```

USES:

```
use modmain
```

DESCRIPTION:

Computes the core radial wavefunctions, eigenvalues and densities. The radial Dirac equation is solved in the spherical part of the Kohn-Sham potential to which the atomic potential has been appended for  $r > R_{\text{MT}}$ . In the case of spin-polarised calculations, and when `spincore` is set to `.true.`, the Dirac equation is solved in the spin-up and -down potentials

created from the Kohn-Sham scalar potential and magnetic field magnitude, with the occupancy divided equally between up and down. The up and down densities determined in this way are added to both the scalar density and the magnetisation in the routine `rhocore`. Note that this procedure is a simple, but inexact, approach to solving the radial Dirac equation in a magnetic field.

#### REVISION HISTORY:

Created April 2003 (JKD)  
Added polarised cores, November 2009 (JKD)

---

### 7.43 `genflu` (Source File: `genflu.f90`)

#### INTERFACE:

```
subroutine genflu(is,l,u,j,f)
```

#### USES:

```
use modldapu
```

#### INPUT/OUTPUT PARAMETERS:

```
is  : species type (in,integer)  
l   : an angular momentum (in,integer)  
u   : parameter U (inout,real)  
j   : parameter J (inout,real)  
f   : Slater parameters (inout,real)
```

#### DESCRIPTION:

Calculate the Slater parameters for LDA+U calculation with different approaches, see *Phys. Rev. B* **80**, 035121 (2009). The relations among Slater and Racah parameters are from E.U. Condon and G.H. Shortley, *The Theory of Atomic Spectra*, The University Press, Cambridge (1935).

#### REVISION HISTORY:

Created July 2008 (Francesco Cricchio)

---

### 7.44 `genflufr` (Source File: `genflufr.f90`)

#### INTERFACE:

```
subroutine genflufr
```

#### USES:

```
use modmain
use modldapu
```

DESCRIPTION:

Generates the radial functions used to calculate the Slater integrals through a Yukawa potential.

REVISION HISTORY:

Created April 2008 from genapwfr (Francesco Cricchio)

---

### 7.45 gengkvec (Source File: gengkvec.f90)

INTERFACE:

```
subroutine gengkvec(ngvec,ivg,vgc,vkl,vkc,gkmax,ngkmax,ngk,igkig,vgkl,vgkc)
```

*INPUT/OUTPUT PARAMETERS:*

```
ngvec  : number of G-vectors (in,integer)
ivg    : G-vector integer coordinates (in,integer(3,ngvec))
vgc    : G-vectors in Cartesian coordinates (in,real(3,ngvec))
vkl    : k-point vector in lattice coordinates (in,real(3))
vkc    : k-point vector in Cartesian coordinates (in,real(3))
gkmax  : G+k-vector cut-off (in,real)
ngkmax : maximum number of G+k-vectors (in,integer)
ngk    : number of G+k-vectors returned (out,integer)
igkig  : index from G+k-vectors to G-vectors (out,integer(ngkmax))
vgkl   : G+k-vectors in lattice coordinates (out,real(3,ngkmax))
vgkc   : G+k-vectors in Cartesian coordinates (out,real(3,ngkmax))
```

DESCRIPTION:

Generates a set of  $\mathbf{G} + \mathbf{k}$ -vectors for the input  $k$ -point with length less than  $gkmax$ .

REVISION HISTORY:

Created April 2003 (JKD)  
Removed spherical coordinate generation, May 2010 (JKD)  
Removed modmain and added arguments, September 2012 (JKD)

---

### 7.46 gengvec (Source File: gengvec.f90)

INTERFACE:

```
subroutine gengvec(ngridg,ngtot,intgv,bvec,gmaxvr,ngvec,ivg,ivgig,igfft,vgc,gc)
```

### INPUT/OUTPUT PARAMETERS:

`ngridg` : G-vector grid sizes (in,integer(3))  
`ngtot` : total number of G-vectors (in,integer)  
`intgv` : integer grid intervals for each direction (in,intgv(2,3))  
`bvec` : reciprocal lattice vectors (in,real(3,3))  
`gmaxvr` : G-vector cut-off (in,gmaxvr)  
`ngvec` : number of G-vectors with  $G < gmaxvr$  (out,integer)  
`ivg` : G-vector integer coordinates (out,integer(3,ngtot))  
`ivgig` : map from (i1,i2,i3) to G-vector index (out,integer(intgv(1,1):  
intgv(2,1),intgv(1,2):intgv(2,2),intgv(1,3):intgv(2,3)))  
`igfft` : map from G-vector array to FFT array (out,integer(ngtot))  
`vgc` : G-vectors in Cartesian coordinates (out,real(3,ngtot))  
`gc` : length of G-vectors (out,real(ngtot))

### DESCRIPTION:

Generates a set of **G**-vectors used for the Fourier transform of the charge density and potential and sorts them according to length. Integers corresponding to the vectors in lattice coordinates are stored, as well as the map from these integer coordinates to the **G**-vector index. A map from the **G**-vector set to the standard FFT array structure is also generated. Finally, the number of **G**-vectors with magnitude less than `gmaxvr` is determined.

### REVISION HISTORY:

Created October 2002 (JKD)  
Removed `modmain` and added arguments, September 2012 (JKD)

---

## 7.47 `genidxlo` (Source File: `genidxlo.f90`)

### INTERFACE:

```
subroutine genidxlo
```

### USES:

```
use modmain
```

### DESCRIPTION:

Generates an index array which maps the local-orbitals in each atom to their locations in the overlap or Hamiltonian matrices. Also finds the total number of local-orbitals.

### REVISION HISTORY:

Created June 2003 (JKD)

---

## 7.48 genlofr (Source File: genlofr.f90)

### INTERFACE:

```
subroutine genlofr
```

### USES:

```
use modmain
```

### DESCRIPTION:

Generates the local-orbital radial functions. This is done by integrating the scalar relativistic Schrödinger equation (or its energy derivatives) at the current linearisation energies using the spherical part of the Kohn-Sham potential. For each local-orbital, a linear combination of `lorbord` radial functions is constructed such that its radial derivatives up to order `lorbord-1` are zero at the muffin-tin radius. This function is normalised and the radial Hamiltonian applied to it. The results are stored in the global array `lofr`.

### REVISION HISTORY:

Created March 2003 (JKD)

Copied to equivalent atoms, February 2010 (A. Kozhevnikov and JKD)

---

## 7.49 genpmat (Source File: genpmat.f90)

### INTERFACE:

```
subroutine genpmat(ngp,igpig,vgpc,wfmt,wfir,pmat)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
ngp      : number of G+p-vectors (in,integer(nspnfv))
igpig    : index from G+p-vectors to G-vectors (in,integer(ngkmax,nspnfv))
vgpc     : G+p-vectors in Cartesian coordinates (in,real(3,ngkmax,nspnfv))
wfmt     : muffin-tin wavefunction in spherical harmonics
          (in,complex(lmmaxvr,nrcmtmax,natmtot,nspinor,nstsv))
wfir     : interstitial wavefunction in plane wave basis
          (in,complex(ngkmax,nspinor,nstsv))
pmat     : momentum matrix elements (out,complex(3,nstsv,nstsv))
```

### DESCRIPTION:

Calculates the momentum matrix elements

$$P_{ij} = \int d^3r \Psi_{i\mathbf{k}}^*(\mathbf{r}) \left( -i\nabla + \frac{1}{4c^2} [\vec{\sigma} \times \nabla V_s(\mathbf{r})] \right) \Psi_{j\mathbf{k}}(\mathbf{r}),$$

where  $V_s$  is the Kohn-Sham effective potential. The second term in the brackets is only calculated if spin-orbit coupling is enabled.

#### REVISION HISTORY:

Created November 2003 (Sharma)  
Fixed bug found by Juergen Spitaler, September 2006 (JKD)  
Added spin-orbit correction, July 2010 (JKD)

---

### 7.50 genppts (Source File: genppts.f90)

#### INTERFACE:

```
subroutine genppts(tfbz,nsym,sym,ngridp,npptnr,epslat,bvec,boxl,nppt,ipmap, &  
  ipmapnr,ivp,vpl,vpc,wppt,wpptnr)
```

#### INPUT/OUTPUT PARAMETERS:

```
tfbz      : .true. if vpl and vpc should be mapped to the first Brillouin zone  
           (in,logical)  
nsym      : number of point group symmetries used for reduction, set to 1 for  
           no reduction (in,integer)  
sym       : symmetry matrices in lattice coordinates (in,integer(3,3,*))  
ngridp    : p-point grid sizes (in,integer(3))  
npptnr    : number of non-reduced p-points: ngridp(1)*ngridp(2)*ngridp(3)  
           (in,integer)  
epslat    : tolerance for determining identical vectors (in,real)  
bvec      : reciprocal lattice vectors (in,real(3,3))  
boxl      : corners of box containing p-points in lattice coordinates, the  
           first vector is the origin (in,real(3,4))  
nppt      : total number of p-points (out,integer)  
ipmap     : map from integer grid to reduced p-point index  
           (out,integer(0:ngridp(1)-1,0:ngridp(2)-1,0:ngridp(3)-1))  
ipmapnr   : map from integer grid to non-reduced p-point index  
           (out,integer(0:ngridp(1)-1,0:ngridp(2)-1,0:ngridp(3)-1))  
ivp       : integer coordinates of the p-points  
           (out,integer(3,ngridp(1)*ngridp(2)*ngridp(3)))  
vpl       : lattice coordinates of each p-point  
           (out,real(3,ngridp(1)*ngridp(2)*ngridp(3)))  
vpc       : Cartesian coordinates of each p-point  
           (out,real(3,ngridp(1)*ngridp(2)*ngridp(3)))  
wppt      : weights of each reduced p-point  
           (out,real(ngridp(1)*ngridp(2)*ngridp(3)))  
wpptnr    : weight of each non-reduced p-point (out,real)
```

#### DESCRIPTION:

This routine is used for generating  $k$ -point or  $q$ -point sets. Since these are stored in global arrays, the points passed to this and other routines are referred to as  $p$ -points. In lattice coordinates, the  $\mathbf{p}$  vectors are given by

$$\mathbf{p} = \begin{pmatrix} \mathbf{B}_2 - \mathbf{B}_1 & \mathbf{B}_3 - \mathbf{B}_1 & \mathbf{B}_4 - \mathbf{B}_1 \end{pmatrix} \begin{pmatrix} i_1/n_1 \\ i_2/n_2 \\ i_3/n_3 \end{pmatrix} + \mathbf{B}_1$$

where  $i_j$  runs from 0 to  $n_j - 1$ , and the  $\mathbf{B}$  vectors define the corners of a box with  $\mathbf{B}_1$  as the origin. If `tfbz` is `.true.` then each `vpl` vector is mapped to the first Brillouin zone. If `tfbz` is `.false.` and `nsym` > 0, then the coordinates of each `vpl` are mapped to the  $[0, 1)$  interval. The  $p$ -point weights are stored in `wppt` and the array `ipmap` contains the map from the integer coordinates to the reduced index.

#### REVISION HISTORY:

Created August 2002 (JKD)  
 Updated April 2007 (JKD)  
 Added mapping to the first Brillouin zone, September 2008 (JKD)  
 Made independent of `modmain`, February 2010 (JKD)

---

## 7.51 genrlm (Source File: genrlm.f90)

### INTERFACE:

```
subroutine genrlm(lmax,tp,rlm)
```

### INPUT/OUTPUT PARAMETERS:

```
lmax : maximum angular momentum (in,integer)
tp   : (theta, phi) coordinates (in,real(2))
rlm  : array of real spherical harmonics (out,real((lmax+1)**2))
```

### DESCRIPTION:

Generates a sequence of real spherical harmonics evaluated at angles  $(\theta, \phi)$  for  $0 < l < l_{\max}$ . The values are returned in a packed array `rlm` indexed with  $j = l(l + 1) + m + 1$ . Real spherical harmonics are defined by

$$R_{lm}(\theta, \phi) = \begin{cases} \sqrt{2} \Re\{Y_{lm}(\theta, \phi)\} & m > 0 \\ \sqrt{2} \Im\{Y_{lm}(\theta, \phi)\} & m < 0, \\ \Re\{Y_{lm}(\theta, \phi)\} & m = 0 \end{cases}$$

where  $Y_{lm}$  are the complex spherical harmonics. These functions are orthonormal and complete and may be used for expanding real-valued functions on the sphere. This routine is numerically stable and accurate to near machine precision for  $l \leq 50$ . See routine `genylm`.

### REVISION HISTORY:

Created March 2004 (JKD)

---

## 7.52 genrmesh (Source File: genrmesh.f90)

### INTERFACE:

```
subroutine genrmesh
```

### USES:

```
use modmain  
use modvars
```

### DESCRIPTION:

Generates the coarse and fine radial meshes for each atomic species in the crystal. Also determines which points are in the inner part of the muffin-tin using the value of `fracinr`.

### REVISION HISTORY:

Created September 2002 (JKD)

---

## 7.53 gensdmat (Source File: gensdmat.f90)

### INTERFACE:

```
subroutine gensdmat(evecsv,sdmat)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
evecsv : second-variational eigenvectors (in,complex(nstsv,nstsv))  
sdmat  : spin density matrices (out,complex(nspinor,nspinor,nstsv))
```

### DESCRIPTION:

Computes the spin density matrices for a set of second-variational states.

### REVISION HISTORY:

Created September 2008 (JKD)

---

## 7.54 gensfacgp (Source File: gensfacgp.f90)

### INTERFACE:

```
subroutine gensfacgp(ngp,vgpc,ld,sfacgp)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
ngp      : number of G+p-vectors (in,integer)
vgpc     : G+p-vectors in Cartesian coordinates (in,real(3,*))
ld       : leading dimension (in,integer)
sfacgp   : structure factors of G+p-vectors (out,complex(ld,natmtot))
```

### DESCRIPTION:

Generates the atomic structure factors for a set of  $\mathbf{G} + \mathbf{p}$ -vectors:

$$S_{\alpha}(\mathbf{G} + \mathbf{p}) = \exp(i(\mathbf{G} + \mathbf{p}) \cdot \mathbf{r}_{\alpha}),$$

where  $\mathbf{r}_{\alpha}$  is the position of atom  $\alpha$ .

### REVISION HISTORY:

Created January 2003 (JKD)

---

## 7.55 genshtmat (Source File: genshtmat.f90)

### INTERFACE:

```
subroutine genshtmat
```

### USES:

```
use modmain
```

### DESCRIPTION:

Generates the forward and backward spherical harmonic transformation (SHT) matrices using the spherical covering set produced by the routine `sphcover`. These matrices are used to transform a function between its  $(l, m)$ -expansion coefficients and its values at the  $(\theta, \phi)$  points on the sphere.

### REVISION HISTORY:

Created April 2003 (JKD)

---

## 7.56 genspchi0 (Source File: genspchi0.f90)

INTERFACE:

```
subroutine genspchi0(ikp,scsr,vqpl,gqc,ylmq,sfacgq,chi0)
```

USES:

```
use modmain
```

INPUT/OUTPUT PARAMETERS:

```
ikp      : k-point from non-reduced set (in,integer)
scsr     : scissor correction (in,real)
vqpl     : input q-point in lattice coordinates (in,real(3))
gqc      : length of G+q-vectors (in,real(ngrf))
ylmq     : spherical harmonics of the G+q-vectors (in,complex(lmmxvr,ngrf))
sfacgq   : structure factors of G+q-vectors (in,complex(ngrf,natmtot))
chi0     : spin-dependent Kohn-Sham response function in G-space
           (out,complex(ngrf,4,ngrf,4,nwrf))
```

DESCRIPTION:

Computes the spin-dependent Kohn-Sham response function:

$$\begin{aligned}\chi_{\alpha\beta,\alpha'\beta'}(\mathbf{r},\mathbf{r}',\omega) &\equiv \frac{\delta\rho_{\alpha\beta}(\mathbf{r},\omega)}{\delta v_{\alpha'\beta'}(\mathbf{r}',\omega)} \\ &= \frac{1}{N_k} \sum_{i\mathbf{k},j\mathbf{k}'} (f_{i\mathbf{k}} - f_{j\mathbf{k}'}) \frac{\langle i\mathbf{k}|\hat{\rho}_{\beta\alpha}(\mathbf{r})|j\mathbf{k}'\rangle \langle j\mathbf{k}'|\hat{\rho}_{\alpha'\beta'}(\mathbf{r}')|i\mathbf{k}\rangle}{w + (\varepsilon_{i\mathbf{k}} - \varepsilon_{j\mathbf{k}'}) + i\eta},\end{aligned}$$

where  $\alpha$  and  $\beta$  are spin-coordinates,  $N_k$  is the number of  $k$ -points,  $f_{i\mathbf{k}}$  are the occupancies,  $v$  is the Kohn-Sham potential and  $\hat{\rho}$  is the spin-density operator. With translational symmetry in mind, we adopt the following convention for its Fourier transform:

$$\chi_{\alpha\beta,\alpha'\beta'}(\mathbf{G},\mathbf{G}',\mathbf{q},\omega) = \frac{1}{\Omega} \int d^3r d^3r' e^{-i(\mathbf{G}+\mathbf{q})\cdot\mathbf{r}} e^{i(\mathbf{G}'+\mathbf{q})\cdot\mathbf{r}'} \chi_{\alpha\beta,\alpha'\beta'}(\mathbf{r},\mathbf{r}',\omega).$$

Let

$$Z_{i\mathbf{k},j\mathbf{k}+\mathbf{q}}^{\alpha\beta}(\mathbf{G}) \equiv \int d^3r e^{i(\mathbf{G}+\mathbf{q})\cdot\mathbf{r}} \varphi_{j\mathbf{k}+\mathbf{q},\alpha}^*(\mathbf{r}) \varphi_{i\mathbf{k},\beta}(\mathbf{r})$$

then the response function in  $G$ -space can be written

$$\chi_{\alpha\beta,\alpha'\beta'}(\mathbf{G},\mathbf{G}',\mathbf{q},\omega) = \frac{1}{N_k\Omega} \sum_{i\mathbf{k},j\mathbf{k}+\mathbf{q}} (f_{i\mathbf{k}} - f_{j\mathbf{k}}) \frac{[Z_{i\mathbf{k},j\mathbf{k}+\mathbf{q}}^{\alpha\beta}(\mathbf{G})]^* Z_{i\mathbf{k},j\mathbf{k}+\mathbf{q}}^{\alpha'\beta'}(\mathbf{G}')}{w + (\varepsilon_{i\mathbf{k}} - \varepsilon_{j\mathbf{k}+\mathbf{q}}) + i\eta}.$$

REVISION HISTORY:

Created March 2012 (SS and JKD)

## 7.57 genveelu (Source File: genveelu.f90)

### INTERFACE:

```
subroutine genveelu(is,l,u,j,vee)
```

### USES:

```
use modmain  
use modldapu
```

### INPUT/OUTPUT PARAMETERS:

```
is   : species type (in,integer)  
l    : an angular momentum (in,integer)  
u    : parameter U (inout,real)  
j    : parameter J (inout,real)  
vee  : Coulomb matrix elements (out,real(-lmaxlu:lmaxlu,-lmaxlu:lmaxlu,  
    -lmaxlu:lmaxlu,-lmaxlu:lmaxlu))
```

### DESCRIPTION:

Calculates the Coulomb matrix elements, see *Phys. Rev. B* **52**, 5467 (1995).

### REVISION HISTORY:

```
Created November 2007 (FC,KD,FB,LN)  
Modified July 2009 (FC)
```

---

## 7.58 genvmatlu (Source File: genvmatlu.f90)

### INTERFACE:

```
subroutine genvmatlu
```

### USES:

```
use modmain  
use modldapu
```

### DESCRIPTION:

Calculate the LDA+U potential, see *Phys. Rev. B* **52**, 5467 (1995) and *Phys. Rev. B* **80**, 035121 (2009).

### REVISION HISTORY:

```
Created November 2007 (FC,FB,LN,JKD)
```

---

## 7.59 genvnlijji (Source File: genvnlijji.f90)

### INTERFACE:

```
subroutine genvnlijji(ikp,vnlijji)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
ikp      : k-point from non-reduced set (in,integer)  
vnlijji : non-local Coulomb matrix elements (out,real(nstsv,nstsv,nkpt))
```

### DESCRIPTION:

Calculates the non-local Coulomb matrix elements of the type  $(i - jj - i)$ .

### REVISION HISTORY:

Created 2008 (Sharma)

---

## 7.60 genvnlijjk (Source File: genvnlijjk.f90)

### INTERFACE:

```
subroutine genvnlijjk(ikp,vnlijjk)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
ikp      : k-point from non-reduced set (in,integer)  
vnlijjk : non-local Coulomb matrix elements  
          (out,complex(nstsv,nstsv,nstsv,nkpt))
```

### DESCRIPTION:

Calculates non-local Coulomb matrix elements of the type  $(i - jj - k)$ .

### REVISION HISTORY:

Created 2008 (Sharma)

---

## 7.61 genvsig (Source File: genvsig.f90)

### INTERFACE:

```
subroutine genvsig
```

### USES:

```
use modmain
```

### DESCRIPTION:

Generates the Fourier transform of the Kohn-Sham effective potential in the interstitial region. The potential is first multiplied by the characteristic function which zeros it in the muffin-tins. See routine `gencfun`.

### REVISION HISTORY:

Created January 2004 (JKD)

---

## 7.62 genwfsv (Source File: genwfsv.f90)

### INTERFACE:

```
subroutine genwfsv(tsh,tgp,tocc,ngp,igpig,occsvp,apwalm,vecfv,vecsv,wfmt, &  
ld,wfir)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
tsh   : .true. if wfmt should be in spherical harmonic basis (in,logical)  
tgp   : .true. if wfir should be in G+p-space, otherwise in real-space  
       (in,logical)  
tocc  : .true. if only occupied wavefunctions are required (in,logical)  
ngp   : number of G+p-vectors (in,integer(nspnfv))  
igpig : index from G+p-vectors to G-vectors (in,integer(ngkmax,nspnfv))  
occsvp : second-variational occupancy for every state (in,real(nstsv))  
apwalm : APW matching coefficients  
       (in,complex(ngkmax,apwordmax,lmmxapw,natmtot,nspnfv))  
vecfv : first-variational eigenvectors (in,complex(nmatmax,nstfv,nspnfv))  
vecsv : second-variational eigenvectors (in,complex(nstsv,nstsv))  
wfmt  : muffin-tin part of the wavefunctions for every state in spherical  
       coordinates (out,complex(lmmxvr,nrcmtmax,natmtot,nspinor,nstsv))  
ld    : leading dimension, at least ngkmax if tgp is .true. and ngtot if  
       tgp is .false. (in,integer)  
wfir  : interstitial part of the wavefunctions for every state  
       (out,complex(ld,nspinor,nstsv))
```

## DESCRIPTION:

Calculates the second-variational spinor wavefunctions in both the muffin-tin and interstitial regions for every state of a particular  $k$ -point. The wavefunctions in both regions are stored on a real-space grid. A coarse radial mesh is assumed in the muffin-tins with with angular momentum cut-off of `lmaxvr`. If `tocc` is `.true.`, then only those states with occupancy greater than `epsocc` are calculated.

## REVISION HISTORY:

Created November 2004 (Sharma)  
Updated for spin-spirals, June 2010 (JKD)

---

## 7.63 genwiq2 (Source File: genwiq2.f90)

### INTERFACE:

```
subroutine genwiq2
```

### USES:

```
use modmain  
use modtest
```

### DESCRIPTION:

The Fock matrix elements

$$V_{ijk}^{\text{NL}} \equiv \sum_{lk'} \int \frac{\Psi_{ik}^\dagger(\mathbf{r}) \cdot \Psi_{lk'}(\mathbf{r}) \Psi_{lk'}^\dagger(\mathbf{r}') \cdot \Psi_{jk}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}'$$

contain a divergent term in the sum over  $\mathbf{k}'$  which behaves as  $1/q^2$ , where  $\mathbf{q} \equiv \mathbf{k} - \mathbf{k}'$  is in the first Brillouin zone. The resulting convergence with respect to the number of discrete  $q$ -points is very slow. This routine computes the weights

$$w_{\mathbf{q}_i} \equiv \int_{V_i} \frac{1}{q^2} d\mathbf{q}, \quad (1)$$

where the integral is over the small parallelepiped centered on  $\mathbf{q}_i$ , so that integrals over the first Brillouin zone of the form

$$I = \int_{\text{BZ}} \frac{f(\mathbf{q})}{q^2} d\mathbf{q},$$

can be approximated by the sum

$$I \approx \sum_i w_{\mathbf{q}_i} f(\mathbf{q}_i)$$

which converges rapidly with respect to the number of  $q$ -points for smooth functions  $f$ . The integral in (1) is determined by evaluating it numerically on increasingly finer grids and extrapolating to the continuum. Agreement with Mathematica to at least 10 significant figures.

## REVISION HISTORY:

Created August 2004 (JKD,SS)

---

## 7.64 genylm (Source File: genylm.f90)

INTERFACE:

```
subroutine genylm(lmax,tp,ylm)
```

*INPUT/OUTPUT PARAMETERS:*

```
lmax : maximum angular momentum (in,integer)
tp   : (theta, phi) coordinates (in,real(2))
ylm  : array of spherical harmonics (out,complex((lmax+1)**2))
```

DESCRIPTION:

Generates a sequence of spherical harmonics, including the Condon-Shortley phase, evaluated at angles  $(\theta, \phi)$  for  $0 < l < l_{\max}$ . The values are returned in a packed array `ylm` indexed with  $j = l(l+1) + m + 1$ . The algorithm of Masters and Richards-Dinger is used, *Geophys. J. Int.* **135**, 307 (1998). This routine is numerically stable and accurate to near machine precision for  $l \leq 50$ .

REVISION HISTORY:

```
Created March 2004 (JKD)
Improved stability, December 2005 (JKD)
```

---

## 7.65 genylmg (Source File: genylmg.f90)

INTERFACE:

```
subroutine genylmg
```

*USES:*

```
use modmain
```

DESCRIPTION:

Generates a set of spherical harmonics,  $Y_{lm}(\hat{\mathbf{G}})$ , with angular momenta up to `lmaxvr` for the set of  $\mathbf{G}$ -vectors.

REVISION HISTORY:

```
Created June 2003 (JKD)
```

---

## 7.66 genzrho (Source File: genzrho.f90)

### INTERFACE:

```
subroutine genzrho(tsh,tspc,wfmt1,wfmt2,wfir1,wfir2,zrhomt,zrhoir)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
tsh      : .true. if the muffin-tin density is to be in spherical harmonics
           (in,logical)
tspc     : .true. if the density should be contracted over spin (in,logical)
wfmt1    : muffin-tin part of wavefunction 1 in spherical coordinates
           (in,complex(lmmaxvr,nrcmtmax,natmtot,nspinor))
wfmt2    : muffin-tin part of wavefunction 2 in spherical coordinates
           (in,complex(lmmaxvr,nrcmtmax,natmtot,nspinor))
wfir1    : interstitial wavefunction 1 (in,complex(ngtot))
wfir2    : interstitial wavefunction 2 (in,complex(ngtot))
zrhomt   : muffin-tin charge density in spherical harmonics/coordinates
           (out,complex(lmmaxvr,nrcmtmax,natmtot))
zrhoir   : interstitial charge density (out,complex(ngtot))
```

### DESCRIPTION:

Calculates the complex overlap charge density from two input wavefunctions:

$$\rho(\mathbf{r}) \equiv \Psi_1^\dagger(\mathbf{r}) \cdot \Psi_2(\mathbf{r}).$$

Note that the muffin-tin wavefunctions are provided in spherical coordinates and the returned density is either in terms of spherical harmonic coefficients or spherical coordinates when `tsh` is `.true.` or `.false.`, respectively.

### REVISION HISTORY:

Created November 2004 (Sharma)

---

## 7.67 getevecfv (Source File: getevecfv.f90)

### INTERFACE:

```
subroutine getevecfv(vpl,vgpl,vecfv)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

vpl : p-point vector in lattice coordinates (in,real(3))  
vgpl : G+p-vectors in lattice coordinates (out,real(3,ngkmax,nspnfv))  
evecfv : first-variational eigenvectors (out,complex(nmatmax,nstfv,nspnfv))

DESCRIPTION:

Reads in a first-variational eigenvector from file. If the input  $k$ -point,  $\mathbf{p}$ , is not in the reduced set, then the eigenvector of the equivalent point is read in and the required rotation/translation operations applied.

REVISION HISTORY:

Created Feburary 2007 (JKD)  
Fixed transformation error, October 2007 (JKD, Anton Kozhevnikov)  
Fixed l.o. rotation, June 2010 (A. Kozhevnikov)

---

## 7.68 getvnljji (Source File: getvnljji.f90)

INTERFACE:

```
subroutine getvnljji(ikp,vnljji)
```

USES:

```
use modmain
```

INPUT/OUTPUT PARAMETERS:

ikp : k-point from non-reduced set (in,integer)  
vnljji : non-local Coulomb matrix elements (out,real(nstsv,nstsv,nkpt))

DESCRIPTION:

Gets non-local Coulomb matrix elements of the type  $(i - jj - i)$  from the file VNLIJJI.OUT.

REVISION HISTORY:

Created 2009 (Sharma)

---

## 7.69 getvnljjk (Source File: getvnljjk.f90)

INTERFACE:

```
subroutine getvnljjk(ikp,vnljjk)
```

USES:

```
use modmain
```

*INPUT/OUTPUT PARAMETERS:*

ikp : k-point from non-reduced set (in,integer)  
vnlijjk : non-local Coulomb matrix elements  
(out,complex(nstsv,nstsv,nstsv,nkpt))

DESCRIPTION:

Gets Coulomb matrix elements of the type  $(i - jj - k)$  from the file VNLIJJK.OUT.

REVISION HISTORY:

Created 2009 (Sharma)

---

**7.70 ggair\_1 (Source File: ggair\_1.f90)**

INTERFACE:

```
subroutine ggair_1(grho,g2rho,g3rho)
```

*USES:*

```
use modmain
```

DESCRIPTION:

Spin-unpolarised version of ggair\_sp.1.

REVISION HISTORY:

Created November 2009 (JKD)

---

**7.71 ggair\_2a (Source File: ggair\_2a.f90)**

INTERFACE:

```
subroutine ggair_2a(g2rho,gvrho,grho2)
```

*USES:*

```
use modmain
```

DESCRIPTION:

Spin-unpolarised version of ggair\_sp.2a.

REVISION HISTORY:

Created November 2009 (JKD and TMcQ)

---

## 7.72 ggair\_2b (Source File: ggair\_2b.f90)

### INTERFACE:

```
subroutine ggair_2b(g2rho,gvrho,vx,vc,dxdg2,dcdg2)
```

### USES:

```
use modmain
```

### DESCRIPTION:

Spin-unpolarised version of ggair\_sp\_2b.

### REVISION HISTORY:

Created November 2009 (JKD and TMcQ)

---

## 7.73 ggair\_sp\_1 (Source File: ggair\_sp\_1.f90)

### INTERFACE:

```
subroutine ggair_sp_1(rhoup,rhodn,grho,gup,gdn,g2up,g2dn,g3rho,g3up,g3dn)
```

### INPUT/OUTPUT PARAMETERS:

```
rhoup : spin-up density (in,real(ngtot))
rhodn : spin-down density (in,real(ngtot))
grho  : |grad rho| (out,real(ngtot))
gup   : |grad rhoup| (out,real(ngtot))
gdn   : |grad rhodn| (out,real(ngtot))
g2up  : grad^2 rhoup (out,real(ngtot))
g2dn  : grad^2 rhodn (out,real(ngtot))
g3rho : (grad rho).(grad |grad rho|) (out,real(ngtot))
g3up  : (grad rhoup).(grad |grad rhoup|) (out,real(ngtot))
g3dn  : (grad rhodn).(grad |grad rhodn|) (out,real(ngtot))
```

### DESCRIPTION:

Computes  $|\nabla\rho|$ ,  $|\nabla\rho^\uparrow|$ ,  $|\nabla\rho^\downarrow|$ ,  $\nabla^2\rho^\uparrow$ ,  $\nabla^2\rho^\downarrow$ ,  $\nabla\rho\cdot(\nabla|\nabla\rho|)$ ,  $\nabla\rho^\uparrow\cdot(\nabla|\nabla\rho^\uparrow|)$  and  $\nabla\rho^\downarrow\cdot(\nabla|\nabla\rho^\downarrow|)$  for the interstitial charge density, as required by the generalised gradient approximation functionals of type 1 for spin-polarised densities. See routines `potxc` and `modxcifc`.

### REVISION HISTORY:

Created October 2004 (JKD)  
Simplified and improved, October 2009 (JKD)

---

## 7.74 ggair\_sp\_2a (Source File: ggair\_sp\_2a.f90)

### INTERFACE:

```
subroutine ggair_sp_2a(rhoup,rhodn,g2up,g2dn,gvup,gvdn,gup2,gdn2,gupdn)
```

### USES:

```
use modmain
```

### DESCRIPTION:

Computes the interstitial gradients  $\nabla^2\rho^\uparrow$ ,  $\nabla^2\rho^\downarrow$ ,  $\nabla\rho^\uparrow$ ,  $\nabla\rho^\downarrow$ ,  $(\nabla\rho^\uparrow)^2$ ,  $(\nabla\rho^\downarrow)^2$  and  $\nabla\rho^\uparrow \cdot \nabla\rho^\downarrow$ . These are used for GGA functionals of type 2 and meta-GGA. See `ggamt_sp_2a` for details.

### REVISION HISTORY:

Created November 2009 (JKD and TMcQ)

---

## 7.75 ggair\_sp\_2b (Source File: ggair\_sp\_2b.f90)

### INTERFACE:

```
subroutine ggair_sp_2b(g2up,g2dn,gvup,gvdn,vxup,vxdn,vcup,vcdn,dxdgu2,dxdgd2, &  
dxdgud,dcdgu2,dcdgd2,dcdgud)
```

### USES:

```
use modmain
```

### DESCRIPTION:

Post processing step of interstitial gradients for GGA type 2. See routine `ggamt_sp_2a` for full details.

### REVISION HISTORY:

Created November 2009 (JKD and TMcQ)

---

## 7.76 ggamt\_1 (Source File: ggamt\_1.f90)

### INTERFACE:

```
subroutine ggamt_1(ias,grho,g2rho,g3rho)
```

### USES:

```
use modmain
```

DESCRIPTION:

Spin-unpolarised version of `ggamt_sp_1`.

REVISION HISTORY:

Created November 2009 (JKD)

---

**7.77 `ggamt_2a` (Source File: `ggamt_2a.f90`)**

INTERFACE:

```
subroutine ggamt_2a(ias,g2rho,gvrho,grho2)
```

*USES:*

```
use modmain
```

DESCRIPTION:

Spin-unpolarised version of `ggamt_sp_2a`.

REVISION HISTORY:

Created November 2009 (JKD and TMcQ)

---

**7.78 `ggamt_2b` (Source File: `ggamt_2b.f90`)**

INTERFACE:

```
subroutine ggamt_2b(is,g2rho,gvrho,vx,vc,dxdg2,dcdg2)
```

*USES:*

```
use modmain
```

DESCRIPTION:

Spin-unpolarised version of `ggamt_sp_2b`.

REVISION HISTORY:

Created November 2009 (JKD and TMcQ)

---

## 7.79 ggamt\_sp\_1 (Source File: ggamt\_sp\_1.f90)

### INTERFACE:

```
subroutine ggamt_sp_1(is,rhoup,rhodn,grho,gup,gdn,g2up,g2dn,g3rho,g3up,g3dn)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
is      : species number (in,integer)
rhoup   : spin-up density in spherical coordinates (in,real(lmmaxvr,nrmtmax))
rhodn   : spin-down density (in,real(lmmaxvr,nrmtmax))
grho    : |grad rho| (out,real(lmmaxvr,nrmtmax))
gup     : |grad rhoup| (out,real(lmmaxvr,nrmtmax))
gdn     : |grad rhodn| (out,real(lmmaxvr,nrmtmax))
g2up    : grad^2 rhoup (out,real(lmmaxvr,nrmtmax))
g2dn    : grad^2 rhodn (out,real(lmmaxvr,nrmtmax))
g3rho   : (grad rho).(grad |grad rho|) (out,real(lmmaxvr,nrmtmax))
g3up    : (grad rhoup).(grad |grad rhoup|) (out,real(lmmaxvr,nrmtmax))
g3dn    : (grad rhodn).(grad |grad rhodn|) (out,real(lmmaxvr,nrmtmax))
```

### DESCRIPTION:

Computes  $|\nabla\rho|$ ,  $|\nabla\rho^\uparrow|$ ,  $|\nabla\rho^\downarrow|$ ,  $\nabla^2\rho^\uparrow$ ,  $\nabla^2\rho^\downarrow$ ,  $\nabla\rho\cdot(\nabla|\nabla\rho|)$ ,  $\nabla\rho^\uparrow\cdot(\nabla|\nabla\rho^\uparrow|)$  and  $\nabla\rho^\downarrow\cdot(\nabla|\nabla\rho^\downarrow|)$  for a muffin-tin charge density, as required by the generalised gradient approximation functionals of type 1 for spin-polarised densities. The input densities and output gradients are in terms of spherical coordinates. See routines `potxc` and `modxcifc`.

### REVISION HISTORY:

```
Created April 2004 (JKD)
Simplified and improved, October 2009 (JKD)
```

---

## 7.80 ggamt\_sp\_2a (Source File: ggamt\_sp\_2a.f90)

### INTERFACE:

```
subroutine ggamt_sp_2a(is,rhoup,rhodn,g2up,g2dn,gvup,gvdn,gup2,gdn2,gupdn)
```

### USES:

```
use modmain
```

### DESCRIPTION:

Computes the muffin-tin gradients  $\nabla^2\rho^\uparrow$ ,  $\nabla^2\rho^\downarrow$ ,  $\nabla\rho^\uparrow$ ,  $\nabla\rho^\downarrow$ ,  $(\nabla\rho^\uparrow)^2$ ,  $(\nabla\rho^\downarrow)^2$  and  $\nabla\rho^\uparrow \cdot \nabla\rho^\downarrow$ , which are passed in to GGA functional subroutines of type 2. The exchange-correlation energy in these routines has the functional form

$$E_{xc}[\rho^\uparrow, \rho^\downarrow] = \int d^3r \hat{\epsilon}_{xc}(\rho^\uparrow(\mathbf{r}), \rho^\downarrow(\mathbf{r}), (\nabla\rho^\uparrow(\mathbf{r}))^2, (\nabla\rho^\downarrow(\mathbf{r}))^2, \nabla\rho^\uparrow(\mathbf{r}) \cdot \nabla\rho^\downarrow(\mathbf{r})),$$

where  $\hat{\epsilon}_{xc}(\mathbf{r}) = \epsilon_{xc}(\mathbf{r})\rho(\mathbf{r})$  is the xc energy per unit volume, with  $\epsilon_{xc}$  being the xc energy per electron, and  $\rho = \rho^\uparrow + \rho^\downarrow$ . From the gradients above, type 2 GGA routines return  $\epsilon_{xc}$ , but not directly the xc potentials. Instead they generate the derivatives  $\partial\hat{\epsilon}_{xc}/\partial\rho^\uparrow(\mathbf{r})$ ,  $\partial\hat{\epsilon}_{xc}/\partial(\nabla\rho^\uparrow(\mathbf{r}))^2$ , and the same for down spin, as well as  $\partial\hat{\epsilon}_{xc}/\partial(\nabla\rho^\uparrow(\mathbf{r}) \cdot \nabla\rho^\downarrow(\mathbf{r}))$ . In a post-processing step invoked by `ggamt_sp_2b`, integration by parts is used to obtain the xc potential explicitly with

$$V_{xc}^\uparrow(\mathbf{r}) = \frac{\partial\hat{\epsilon}_{xc}}{\partial\rho^\uparrow(\mathbf{r})} - 2 \left( \nabla \frac{\partial\hat{\epsilon}_{xc}}{\partial(\nabla\rho^\uparrow)^2} \right) \cdot \nabla\rho^\uparrow - 2 \frac{\hat{\epsilon}_{xc}}{\partial(\nabla\rho^\uparrow)^2} \nabla^2\rho^\uparrow \\ - \left( \nabla \frac{\hat{\epsilon}_{xc}}{\partial(\nabla\rho^\uparrow \cdot \nabla\rho^\downarrow)} \right) \cdot \nabla\rho^\downarrow - \frac{\partial\hat{\epsilon}_{xc}}{\partial(\nabla\rho^\uparrow \cdot \nabla\rho^\downarrow)} \nabla^2\rho^\downarrow,$$

and similarly for  $V_{xc}^\downarrow$ .

REVISION HISTORY:

Created November 2009 (JKD and TMcQ)

---

## 7.81 `ggamt_sp_2b` (Source File: `ggamt_sp_2b.f90`)

INTERFACE:

```
subroutine ggamt_sp_2b(is, g2up, g2dn, gvup, gvdn, vxup, vxdn, vcup, vcdn, dxdgu2, &
  dxdgd2, dxdgud, dcdgu2, dcdgd2, dcdgud)
```

USES:

```
use modmain
```

DESCRIPTION:

Post processing step of muffin-tin gradients for GGA type 2. See routine `ggamt_sp_2a` for full details.

REVISION HISTORY:

Created November 2009 (JKD and TMcQ)

---

## 7.82 gndstate (Source File: gndstate.f90)

### INTERFACE:

```
subroutine gndstate
```

### USES:

```
use modmain
use modmpi
use modldapu
```

### DESCRIPTION:

Computes the self-consistent Kohn-Sham ground-state. General information is written to the file `INFO.OUT`. First- and second-variational eigenvalues, eigenvectors and occupancies are written to the unformatted files `EVALFV.OUT`, `EVALSV.OUT`, `EVECFV.OUT`, `EVECSV.OUT` and `OCCSV.OUT`. The density, magnetisation, Kohn-Sham potential and magnetic field are written to `STATE.OUT`.

### REVISION HISTORY:

```
Created October 2002 (JKD)
Added MPI, August 2010 (JKD)
```

---

## 7.83 gradrfmt (Source File: gradrfmt.f90)

### INTERFACE:

```
subroutine gradrfmt(nr,nri,r,rfmt,ld,grfmt)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
nr      : number of radial mesh points (in,integer)
nri     : number of points on inner part of muffin-tin (in,integer)
r       : radial mesh (in,real(nr))
rfmt    : real muffin-tin function (in,real(lmmaxvr,nr))
ld      : leading dimension (in,integer)
grfmt   : gradient of rfmt (out,real(lmmaxvr,ld,3))
```

### DESCRIPTION:

Calculates the gradient of a real muffin-tin function. In other words, given the real spherical harmonic expansion coefficients,  $f_{lm}(r)$ , of a function  $f(\mathbf{r})$ , the routine returns  $\mathbf{F}_{lm}$  where

$$\sum_{lm} \mathbf{F}_{lm}(r) R_{lm}(\hat{\mathbf{r}}) = \nabla f(\mathbf{r}),$$

and  $R_{lm}$  is a real spherical harmonic function. This is done by first converting the function to a complex spherical harmonic expansion and then using the routine `gradzfmt`. See routine `genrlm`.

#### REVISION HISTORY:

Created August 2003 (JKD)

---

### 7.84 gradzfmt (Source File: gradzfmt.f90)

#### INTERFACE:

```
subroutine gradzfmt(nr,nri,r,zfmt,ld,gzfmt)
```

#### USES:

```
use modmain
```

#### INPUT/OUTPUT PARAMETERS:

```
nr      : number of radial mesh points (in,integer)
nri     : number of points on inner part of muffin-tin (in,integer)
r       : radial mesh (in,real(nr))
zfmt    : complex muffin-tin function (in,complex(lmmaxvr,nr))
ld      : leading dimension (in,integer)
gzfmt   : gradient of zfmt (out,complex(lmmaxvr,ld,3))
```

#### DESCRIPTION:

Calculates the gradient of a complex muffin-tin function. In other words, given the spherical harmonic expansion coefficients,  $f_{lm}(r)$ , of a function  $f(\mathbf{r})$ , the routine returns  $\mathbf{F}_{lm}$  where

$$\sum_{lm} \mathbf{F}_{lm}(r) Y_{lm}(\hat{\mathbf{r}}) = \nabla f(\mathbf{r}).$$

This is done using the formula (see, for example, V. Devanathan, *Angular Momentum Techniques In Quantum Mechanics*)

$$\begin{aligned} \nabla_{\mu}^s f_{lm}(r) Y_{lm}(\hat{\mathbf{r}}) = & \sqrt{\frac{l+1}{2l+3}} C(l, 1, l+1 | m, \mu, m+\mu) Y_{l+1, m+\mu}(\hat{\mathbf{r}}) \left( \frac{d}{dr} - \frac{l}{r} \right) f_{lm}(r) \\ & - \sqrt{\frac{l}{2l-1}} C(l, 1, l-1 | m, \mu, m+\mu) Y_{l-1, m+\mu}(\hat{\mathbf{r}}) \left( \frac{d}{dr} + \frac{l+1}{r} \right) f_{lm}(r), \end{aligned}$$

where  $C$  are Clebsch-Gordan coefficients and the gradient  $\nabla_{\mu}^s$  is in terms of the spherical unit vectors  $\hat{\mathbf{e}}_{\mu}$ :

$$\hat{\mathbf{e}}_{+1} = -\frac{\hat{\mathbf{x}} + i\hat{\mathbf{y}}}{\sqrt{2}}, \quad \hat{\mathbf{e}}_0 = \hat{\mathbf{z}}, \quad \hat{\mathbf{e}}_{-1} = \frac{\hat{\mathbf{x}} - i\hat{\mathbf{y}}}{\sqrt{2}}.$$

Note that the gradient returned is in terms of the global  $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$  coordinate system.

#### REVISION HISTORY:

Rewritten May 2009 (JKD)

---

## 7.85 gridsize (Source File: gridsize.f90)

INTERFACE:

```
subroutine gridsize(avec,gmaxvr,ngridg,ngtot,intgv)
```

*INPUT/OUTPUT PARAMETERS:*

```
avec   : lattice vectors (in,real(3,3))
gmaxvr : G-vector cut-off (in,real)
ngridg : G-vector grid sizes (out,integer(3))
ngtot  : total number of G-vectors (out,integer)
intgv  : integer grid intervals for each direction (out,integer(2,3))
```

DESCRIPTION:

Finds the **G**-vector grid which completely contains the vectors with  $G < G_{\max}$  and is compatible with the FFT routine. The optimal sizes are given by

$$n_i = \frac{G_{\max} |\mathbf{a}_i|}{\pi} + 1,$$

where  $\mathbf{a}_i$  is the  $i$ th lattice vector.

REVISION HISTORY:

Created July 2003 (JKD)

Removed modmain and added arguments, September 2012 (JKD)

---

## 7.86 hermite (Source File: hermite.f90)

INTERFACE:

```
real(8) function hermite(n,x)
```

*INPUT/OUTPUT PARAMETERS:*

```
n : order of Hermite polynomial (in,integer)
x : real argument (in,real)
```

DESCRIPTION:

Returns the  $n$ th Hermite polynomial. The recurrence relation

$$H_i(x) = 2xH_{i-1}(x) - 2iH_{i-2}(x),$$

with  $H_0 = 1$  and  $H_1 = 2x$ , is used. This procedure is numerically stable and accurate to near machine precision for  $n \leq 20$ .

REVISION HISTORY:

Created April 2003 (JKD)

---

## 7.87 hmlaa (Source File: hmlaa.f90)

### INTERFACE:

```
subroutine hmlaa(ias,ngp,apwalm,ld,h)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
ias   : joint atom and species number (in,integer)
ngp   : number of G+p-vectors (in,integer)
apwalm : APW matching coefficients
        (in,complex(ngkmax,apwordmax,lmmmaxapw,natmtot))
ld    : leading dimension of h (in,integer)
h     : Hamiltonian matrix (inout,complex(*))
```

### DESCRIPTION:

Calculates the APW-APW contribution to the Hamiltonian matrix.

### REVISION HISTORY:

Created October 2002 (JKD)

---

## 7.88 hmlistl (Source File: hmlistl.f90)

### INTERFACE:

```
subroutine hmlistl(ngp,igpig,vgpc,ld,h)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
ngp   : number of G+p-vectors (in,integer)
igpig : index from G+p-vectors to G-vectors (in,integer(ngkmax))
vgpc  : G+p-vectors in Cartesian coordinates (in,real(3,ngkmax))
ld    : leading dimension of h (in,integer)
h     : Hamiltonian matrix (inout,complex(*))
```

### DESCRIPTION:

Computes the interstitial contribution to the Hamiltonian matrix for the APW basis functions. The Hamiltonian is given by

$$H^I(\mathbf{G} + \mathbf{k}, \mathbf{G}' + \mathbf{k}) = \frac{1}{2}(\mathbf{G} + \mathbf{k}) \cdot (\mathbf{G}' + \mathbf{k}) \tilde{\Theta}(\mathbf{G} - \mathbf{G}') + V_s(\mathbf{G} - \mathbf{G}'),$$

where  $V_s$  is the interstitial Kohn-Sham potential and  $\tilde{\Theta}$  is the characteristic function. See routine `gencfun`.

#### REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.89 hmlrad (Source File: hmlrad.f90)

#### INTERFACE:

```
subroutine hmlrad
```

#### USES:

```
use modmain
```

#### DESCRIPTION:

Calculates the radial Hamiltonian integrals of the APW and local-orbital basis functions. In other words, for atom  $\alpha$ , it computes integrals of the form

$$h_{qq';ll'l'm''}^{\alpha} = \begin{cases} \int_0^{R_i} u_{q;l}^{\alpha}(r) H u_{q';l'}^{\alpha}(r) r^2 dr & l'' = 0 \\ \int_0^{R_i} u_{q;l}^{\alpha}(r) V_{l''m''}^{\alpha}(r) u_{q';l'}^{\alpha}(r) r^2 dr & l'' > 0 \end{cases}$$

where  $u_{q;l}^{\alpha}$  is the  $q$ th APW radial function for angular momentum  $l$ ;  $H$  is the Hamiltonian of the radial Schrödinger equation; and  $V_{l''m''}^{\alpha}$  is the muffin-tin Kohn-Sham potential. Similar integrals are calculated for APW-local-orbital and local-orbital-local-orbital contributions.

#### REVISION HISTORY:

Created December 2003 (JKD)

---

### 7.90 i3mdet (Source File: i3mdet.f90)

#### INTERFACE:

```
integer function i3mdet(a)
```

#### INPUT/OUTPUT PARAMETERS:

```
a : input matrix (in,integer(3,3))
```

#### DESCRIPTION:

Returns the determinant of an integer  $3 \times 3$  matrix  $A$ .

#### REVISION HISTORY:

Created October 2004 (JKD)

---

### 7.91 i3minv (Source File: i3minv.f90)

INTERFACE:

```
subroutine i3minv(a,b)
```

*INPUT/OUTPUT PARAMETERS:*

```
  a : input matrix (in,integer(3,3))  
  b : output matrix (in,integer(3,3))
```

DESCRIPTION:

Computes the inverse of a integer  $3 \times 3$  matrix:  $B = A^{-1}$ .

REVISION HISTORY:

Created November 2003 (JKD)

---

### 7.92 i3mtv (Source File: i3mtv.f90)

INTERFACE:

```
subroutine i3mtv(a,x,y)
```

*INPUT/OUTPUT PARAMETERS:*

```
  a : input matrix (in,integer(3,3))  
  x : input vector (in,integer(3))  
  y : output vector (out,integer(3))
```

DESCRIPTION:

Multiplies the transpose of an integer  $3 \times 3$  matrix with a vector.

REVISION HISTORY:

Created April 2007 (JKD)

---

### 7.93 init0 (Source File: init0.f90)

INTERFACE:

```
subroutine init0
```

*USES:*

```
use modmain
use modxcifc
use modldapu
use modtest
use modvars
```

DESCRIPTION:

Performs basic consistency checks as well as allocating and initialising global variables not dependent on the  $k$ -point set.

REVISION HISTORY:

Created January 2004 (JKD)

---

## 7.94 `init1` (Source File: `init1.f90`)

INTERFACE:

```
subroutine init1
```

*USES:*

```
use modmain
use modldapu
use modtest
use modvars
```

DESCRIPTION:

Generates the  $k$ -point set and then allocates and initialises global variables which depend on the  $k$ -point set.

REVISION HISTORY:

Created January 2004 (JKD)

---

## 7.95 `linengy` (Source File: `linengy.f90`)

INTERFACE:

```
subroutine linengy
```

*USES:*

```
use modmain
use modmpi
```

## DESCRIPTION:

Calculates the new linearisation energies for both the APW and local-orbital radial functions. See the routine `findband`.

## REVISION HISTORY:

Created May 2003 (JKD)

---

## 7.96 `lopzflm` (Source File: `lopzflm.f90`)

### INTERFACE:

```
subroutine lopzflm(lmax,zflm,ld,zlflm)
```

### INPUT/OUTPUT PARAMETERS:

`lmax` : maximum angular momentum (in,integer)  
`zflm` : coefficients of input spherical harmonic expansion  
(in,complex((lmax+1)\*\*2))  
`ld` : leading dimension (in,integer)  
`zlflm` : coefficients of output spherical harmonic expansion  
(out,complex(ld,3))

### DESCRIPTION:

Applies the angular momentum operator  $\hat{L}$  to a function expanded in terms of complex spherical harmonics. This makes use of the identities

$$\begin{aligned}(L_x + iL_y)Y_{lm}(\theta, \phi) &= \sqrt{(l-m)(l+m+1)}Y_{l,m+1}(\theta, \phi) \\ (L_x - iL_y)Y_{lm}(\theta, \phi) &= \sqrt{(l+m)(l-m+1)}Y_{l,m-1}(\theta, \phi) \\ L_z Y_{lm}(\theta, \phi) &= mY_{lm}(\theta, \phi).\end{aligned}$$

## REVISION HISTORY:

Created March 2004 (JKD)

---

## 7.97 `match` (Source File: `match.f90`)

### INTERFACE:

```
subroutine match(ngp,gpc,tpgpc,sfacgp,apwalm)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

ngp : number of G+p-vectors (in,integer)  
gpc : length of G+p-vectors (in,real(ngkmax))  
tpgpc : (theta, phi) coordinates of G+p-vectors (in,real(2,ngkmax))  
sfacgp : structure factors of G+p-vectors (in,complex(ngkmax,natmtot))  
apwalm : APW matching coefficients  
(out,complex(ngkmax,apwordmax,lmmmaxapw,natmtot))

### DESCRIPTION:

Computes the  $(\mathbf{G} + \mathbf{p})$ -dependent matching coefficients for the APW basis functions. Inside muffin-tin  $\alpha$ , the APW functions are given by

$$\phi_{\mathbf{G}+\mathbf{p}}^{\alpha}(\mathbf{r}) = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l \sum_{j=1}^{M_l^{\alpha}} A_{jlm}^{\alpha}(\mathbf{G} + \mathbf{p}) u_{jl}^{\alpha}(r) Y_{lm}(\hat{\mathbf{r}}),$$

where  $A_{jlm}^{\alpha}(\mathbf{G} + \mathbf{p})$  is the matching coefficient,  $M_l^{\alpha}$  is the order of the APW and  $u_{jl}^{\alpha}$  is the radial function. In the interstitial region, an APW function is a plane wave,  $\exp(i(\mathbf{G} + \mathbf{p}) \cdot \mathbf{r})/\sqrt{\Omega}$ , where  $\Omega$  is the unit cell volume. Ensuring continuity up to the  $(M_l^{\alpha} - 1)$ th derivative across the muffin-tin boundary therefore requires that the matching coefficients satisfy

$$\sum_{j=1}^{M_l^{\alpha}} D_{ij} A_{jlm}^{\alpha}(\mathbf{G} + \mathbf{p}) = b_i,$$

where

$$D_{ij} = \left. \frac{d^{i-1} u_{jl}^{\alpha}(r)}{dr^{i-1}} \right|_{r=R_{\alpha}}$$

and

$$b_i = \frac{4\pi i^l}{\sqrt{\Omega}} |\mathbf{G} + \mathbf{p}|^{i-1} j_l^{(i-1)}(|\mathbf{G} + \mathbf{p}| R_{\alpha}) \exp(i(\mathbf{G} + \mathbf{p}) \cdot \mathbf{r}_{\alpha}) Y_{lm}^*(\widehat{\mathbf{G} + \mathbf{p}}),$$

with  $\mathbf{r}_{\alpha}$  the atomic position and  $R_{\alpha}$  the muffin-tin radius. See routine `wavfemt`.

### REVISION HISTORY:

Created April 2003 (JKD)  
Fixed documentation, June 2006 (JKD)

---

## 7.98 mixadapt (Source File: mixadapt.f90)

### INTERFACE:

```
subroutine mixadapt(iscl,beta0,betamax,n,nu,mu,beta,f,d)
```

### INPUT/OUTPUT PARAMETERS:

```

iscl      : self-consistent loop number (in,integer)
beta0     : mixing parameter (in,real)
betamax   : maximum mixing parameter (in,real)
n         : vector length (in,integer)
nu        : current output vector as well as the next input vector of the
           self-consistent loop (inout,real(n))
mu        : used internally (inout,real(n))
beta      : used internally (inout,real(n))
f         : used internally (inout,real(n))
d         : RMS difference between old and new output vectors (out,real)

```

DESCRIPTION:

Given the input vector  $\mu^i$  and output vector  $\nu^i$  of the  $i$ th self-consistent loop, this routine generates the next input vector to the loop using an adaptive mixing scheme. The  $j$ th component of the output vector is mixed with a fraction of the same component of the input vector:

$$\mu_j^{i+1} = \beta_j^i \nu_j^i + (1 - \beta_j^i) \mu_j^i,$$

where  $\beta_j^{i+1} = \beta_j^i + \beta_0$  if  $f_j^i \equiv \nu_j^i - \mu_j^i$  does not change sign between loops. If  $f_j^i$  does change sign, then  $\beta_j^{i+1} = (\beta_j^i + \beta_0)/2$ . Note that the array `nu` serves for both input and output, and the arrays `mu`, `beta` and `f` are used internally and should not be changed between calls. The routine is thread-safe so long as each thread has its own independent work arrays. Complex arrays may be passed as real arrays with  $n$  doubled.

REVISION HISTORY:

```

Created March 2003 (JKD)
Modified, September 2008 (JKD)
Modified, August 2011 (JKD)

```

## 7.99 mixpack (Source File: mixpack.f90)

INTERFACE:

```
subroutine mixpack(tpack,n,v)
```

*USES:*

```

use modmain
use modldapu

```

*INPUT/OUTPUT PARAMETERS:*

```

tpack : .true. for packing, .false. for unpacking (in,logical)
n      : total number of real values stored (out,integer)
v      : packed potential (inout,real(*))

```

DESCRIPTION:

Packs/unpacks the muffin-tin and interstitial parts of the Kohn-Sham potential and magnetic field into/from the single array *v*. This array can then be passed directly to the mixing routine. See routine *rpack*.

REVISION HISTORY:

Created June 2003 (JKD)

---

### 7.100 mixpulay (Source File: mixpulay.f90)

INTERFACE:

```
subroutine mixpulay(iscl,n,msd,nu,mu,f,d)
```

INPUT/OUTPUT PARAMETERS:

```
iscl : self-consistent loop number (in,integer)
n    : vector length (in,integer)
msd  : maximum subspace dimension (in,integer)
nu   : current output vector as well as the next input vector of the
      : self-consistent loop (inout,real(n))
mu   : used internally (inout,real(n,msd))
f    : used internally (inout,real(n,msd))
d    : RMS difference between old and new output vectors (out,real)
```

DESCRIPTION:

Pulay's mixing scheme which uses direct inversion in the iterative subspace (DIIS). See *Chem. Phys. Lett.* **73**, 393 (1980).

REVISION HISTORY:

Created October 2008 (S. Suehara, NIMS)  
Modified, October 2008 (JKD)

---

### 7.101 randomu (Source File: modrandom.f90)

INTERFACE:

```
real(8) function randomu()
```

DESCRIPTION:

Generates random numbers with a uniform distribution using the fifth-order multiple recursive generator of P. L'Ecuyer, F. Blouin, and R. Coutre, *ACM Trans. Modeling Comput. Simulation* **3**, 87 (1993). The sequence of numbers  $r_i$  is produced from

$$x_i = (a_1 x_{i-1} + a_5 x_{i-5}) \pmod{m}$$

with  $r_i = x_i/m$ . The period is about  $2^{155}$ .

REVISION HISTORY:

Created January 2012 (JKD)

---

## 7.102 xcifc (Source File: modxcifc.f90)

INTERFACE:

```
subroutine xcifc(xctype,n,c_tb09,rho,rhoup,rhodn,grho,gup,gdn,g2rho,g2up,g2dn, &
  g3rho,g3up,g3dn,grho2,gup2,gdn2,gupdn,tau,tauup,taudn,ex,ec,vx,vc,vxup,vxdn, &
  vcup,vcdn,dxdg2,dxdgu2,dxdgd2,dxdgud,dcdg2,dcdgu2,dcdgd2,dcdgud)
```

INPUT/OUTPUT PARAMETERS:

```
xctype : type of exchange-correlation functional (in,integer(3))
n       : number of density points (in,integer)
c_tb09 : Tran-Blaha '09 constant c (in,real,optional)
rho     : spin-unpolarised charge density (in,real(n),optional)
rhoup  : spin-up charge density (in,real(n),optional)
rhodn  : spin-down charge density (in,real(n),optional)
grho   : |grad rho| (in,real(n),optional)
gup    : |grad rhoup| (in,real(n),optional)
gdn    : |grad rhodn| (in,real(n),optional)
g2rho  : grad^2 rho (in,real(n),optional)
g2up   : grad^2 rhoup (in,real(n),optional)
g2dn   : grad^2 rhodn (in,real(n),optional)
g3rho  : (grad rho).(grad |grad rho|) (in,real(n),optional)
g3up   : (grad rhoup).(grad |grad rhoup|) (in,real(n),optional)
g3dn   : (grad rhodn).(grad |grad rhodn|) (in,real(n),optional)
grho2  : |grad rho|^2 (in,real(n),optional)
gup2   : |grad rhoup|^2 (in,real(n),optional)
gdn2   : |grad rhodn|^2 (in,real(n),optional)
gupdn  : (grad rhoup).(grad rhodn) (in,real(n),optional)
tau    : kinetic energy density (in,real(n),optional)
tauup  : spin-up kinetic energy density (in,real(n),optional)
taudn  : spin-down kinetic energy density (in,real(n),optional)
ex     : exchange energy density (out,real(n),optional)
ec     : correlation energy density (out,real(n),optional)
vx     : spin-unpolarised exchange potential (out,real(n),optional)
vc     : spin-unpolarised correlation potential (out,real(n),optional)
```

```

vxup   : spin-up exchange potential (out,real(n),optional)
vxdn   : spin-down exchange potential (out,real(n),optional)
vcup   : spin-up correlation potential (out,real(n),optional)
vcdn   : spin-down correlation potential (out,real(n),optional)
dxdg2  : de_x/d(|grad rho|^2) (out,real(n),optional)
dxdgu2 : de_x/d(|grad rhoup|^2) (out,real(n),optional)
dxdgd2 : de_x/d(|grad rhodn|^2) (out,real(n),optional)
dxdgud : de_x/d((grad rhoup).(grad rhodn)) (out,real(n),optional)
dcdg2  : de_c/d(|grad rho|^2) (out,real(n),optional)
dcdgu2 : de_c/d(|grad rhoup|^2) (out,real(n),optional)
dcdgd2 : de_c/d(|grad rhodn|^2) (out,real(n),optional)
dcdgud : de_c/d((grad rhoup).(grad rhodn)) (out,real(n),optional)

```

DESCRIPTION:

Interface to the exchange-correlation routines. In the most general case (meta-GGA), the exchange-correlation energy is given by

$$E_{xc}[\rho^\uparrow, \rho^\downarrow] = \int d^3r \rho(\mathbf{r}) \varepsilon_{xc}(\rho^\uparrow, \rho^\downarrow, |\nabla\rho|, |\nabla\rho^\uparrow|, |\nabla\rho^\downarrow|, \nabla^2\rho^\uparrow, \nabla^2\rho^\downarrow, \tau),$$

where  $\rho(\mathbf{r}) = \rho^\uparrow(\mathbf{r}) + \rho^\downarrow(\mathbf{r})$  is the density;

$$\tau(\mathbf{r}) \equiv \sum_{i \text{ occ}} \nabla\psi(\mathbf{r}) \cdot \nabla\psi(\mathbf{r})$$

is twice the spin-contracted kinetic energy density; and  $\varepsilon_{xc}$  is the exchange-correlation energy per electron.

REVISION HISTORY:

Created October 2002 (JKD)

### 7.103 getxcdata (Source File: modxcifc.f90)

INTERFACE:

```
subroutine getxcdata(xctype,xcdescr,xcspin,xcgrad,hybrid,hybridc)
```

INPUT/OUTPUT PARAMETERS:

```

xctype  : type of exchange-correlation functional (in,integer(3))
xcdescr : description of functional (out,character(256))
xcspin  : spin treatment (out,integer)
xcgrad  : gradient treatment (out,integer)
hybrid  : .true. if functional a hybrid (out,logical)
hybridc : hybrid exact exchange mixing coefficient (out,real(8))

```

## DESCRIPTION:

Returns data on the exchange-correlation functional labeled by `xctype`. The character array `xcdescr` contains a short description of the functional including journal references. The variable `xcspin` is set to 1 or 0 for spin-polarised or -unpolarised functionals, respectively. For functionals which require the gradients of the density `xcgrad` is set to 1, otherwise it is set to 0.

## REVISION HISTORY:

Created October 2002 (JKD)

---

### 7.104 moment (Source File: moment.f90)

#### INTERFACE:

```
subroutine moment
```

#### USES:

```
use modmain  
use modtest
```

#### DESCRIPTION:

Computes the muffin-tin, interstitial and total moments by integrating the magnetisation.

#### REVISION HISTORY:

Created January 2005 (JKD)

---

### 7.105 mossbauer (Source File: mossbauer.f90)

#### INTERFACE:

```
subroutine mossbauer
```

#### USES:

```
use modmain
```

#### DESCRIPTION:

Computes the contact charge density and contact magnetic hyperfine field for each atom and outputs the data to the file `MOSSBAUER.OUT`. The Thomson radius,  $R_T = Z/c^2$ , is used for determining the contact moment  $m_c$ , the relation of which to field strength is via Fermi's formula

$$B_c = \frac{8\pi}{3} \mu_B m_c,$$

where the orbital and dipolar contributions are neglected. See S. Blügel, H. Akai, R. Zeller, and P. H. Dederichs, *Phys. Rev. B* **35**, 3271 (1987). See also `radnucl`.

REVISION HISTORY:

Created May 2004 (JKD)

---

### 7.106 `mtdmin` (Source File: `mtdmin.f90`)

INTERFACE:

```
subroutine mtdmin(is,js,dmin)
```

*USES:*

```
use modmain
```

*INPUT/OUTPUT PARAMETERS:*

```
is, js : species numbers (out,integer)
dmin   : minimum distance between muffin-tin surfaces (out,real)
```

DESCRIPTION:

Finds the atomic species pair for which the distance between the muffin-tin surfaces is a minimum. This distance may be negative if the muffin-tins overlap.

REVISION HISTORY:

Created October 2011 (JKD)

---

### 7.107 `nfftifc` (Source File: `nfftifc.f90`)

INTERFACE:

```
subroutine nfftifc(n)
```

*INPUT/OUTPUT PARAMETERS:*

```
n : required/available grid size (inout,integer)
```

DESCRIPTION:

Interface to the grid requirements of the fast Fourier transform routine. Most routines restrict  $n$  to specific prime factorisations. This routine returns the next largest grid size allowed by the FFT routine.

REVISION HISTORY:

Created October 2002 (JKD)

---

## 7.108 nonlinopt (Source File: nonlinopt.f90)

### INTERFACE:

```
subroutine nonlinopt
```

### USES:

```
use modmain
```

### DESCRIPTION:

Calculates susceptibility tensor for non-linear optical second-harmonic generation (SHG). The terms (ztn) are numbered according to Eqs. (49)-(51) of the article *Physica Scripta* **T109**, 128 (2004). Other good references are *Phys. Rev. B* **48**, 11705 (1993) and *Phys. Rev. B* **53**, 10751 (1996).

### REVISION HISTORY:

Rewrote earlier version, June 2010 (Sharma)

---

## 7.109 occupy (Source File: occupy.f90)

### INTERFACE:

```
subroutine occupy
```

### USES:

```
use modmain  
use modtest
```

### DESCRIPTION:

Finds the Fermi energy and sets the occupation numbers for the second-variational states using the routine `fermi`.

### REVISION HISTORY:

Created February 2004 (JKD)  
Added gap estimation, November 2009 (F. Cricchio)  
Added adaptive smearing width, April 2010 (T. Bjorkman)

---

## 7.110 `olpistl` (Source File: `olpistl.f90`)

### INTERFACE:

```
subroutine olpistl(ngp,igpig,ld,o)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
ngp  : number of G+p-vectors (in,integer)
igpig : index from G+p-vectors to G-vectors (in,integer(ngkmax))
ld    : leading dimension of o (in,integer)
o     : overlap matrix (inout,complex(*))
```

### DESCRIPTION:

Computes the interstitial contribution to the overlap matrix for the APW basis functions. The overlap is given by

$$O^I(\mathbf{G} + \mathbf{k}, \mathbf{G}' + \mathbf{k}) = \tilde{\Theta}(\mathbf{G} - \mathbf{G}'),$$

where  $\tilde{\Theta}$  is the characteristic function. See routine `gencfun`.

### REVISION HISTORY:

Created April 2003 (JKD)

---

## 7.111 `olprad` (Source File: `olprad.f90`)

### INTERFACE:

```
subroutine olprad
```

### USES:

```
use modmain
```

### DESCRIPTION:

Calculates the radial overlap integrals of the APW and local-orbital basis functions. In other words, for atom  $\alpha$ , it computes integrals of the form

$$o_{qp}^{\alpha} = \int_0^{R_i} u_{q;l}^{\alpha}(r) v_p^{\alpha}(r) r^2 dr$$

and

$$o_{pp'}^{\alpha} = \int_0^{R_i} v_p^{\alpha}(r) v_{p'}^{\alpha}(r) r^2 dr, \quad l_p = l_{p'}$$

where  $u_{q;l}^{\alpha}$  is the  $q$ th APW radial function for angular momentum  $l$ ; and  $v_p^{\alpha}$  is the  $p$ th local-orbital radial function and has angular momentum  $l_p$ .

### REVISION HISTORY:

Created November 2003 (JKD)

---

### 7.112 pade (Source File: pade.f90)

#### INTERFACE:

```
subroutine pade(nin,zin,uin,nout,zout,uout)
```

#### INPUT/OUTPUT PARAMETERS:

```
  nin : number of input points (in,integer)
  zin : input points (in,complex(nin))
  uin : input function values (in,complex(nin))
  nout : number of output points (in,integer)
  zout : output points (in,complex(nout))
  uout : output function values (out,complex(nout))
```

#### DESCRIPTION:

Calculates a Padé approximant of a function, given the function evaluated on a set of points in the complex plane. The function is returned for a set of complex output points. The algorithm from H. J. Vidberg and J. W. Serene *J. Low Temp. Phys.* 29, 179 (1977) is used.

#### REVISION HISTORY:

Created December 2010 (Antonio Sanna)

---

### 7.113 plot1d (Source File: plot1d.f90)

#### INTERFACE:

```
subroutine plot1d(fnum1,fnum2,nf,rfmt,rfir)
```

#### USES:

```
use modmain
```

#### INPUT/OUTPUT PARAMETERS:

```
  fnum1 : plot file number (in,integer)
  fnum2 : vertex location file number (in,integer)
  nf     : number of functions (in,integer)
  rfmt  : real muffin-tin function (in,real(lmmaxvr,nrmtmax,natmtot,nf))
  rfir  : real interstitial function (in,real(ngtot,nf))
```

#### DESCRIPTION:

Produces a 1D plot of the real functions contained in arrays `rfmt` and `rfir` along the lines connecting the vertices in the global array `vvlp1d`. See routine `rfarray`.

#### REVISION HISTORY:

Created June 2003 (JKD)

---

## 7.114 plot2d (Source File: plot2d.f90)

### INTERFACE:

```
subroutine plot2d(fnum,nf,rfmt,rfir)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
fnum : plot file number (in,integer)
nf    : number of functions (in,integer)
rfmt  : real muffin-tin function (in,real(lmmaxvr,nrmtmax,natmtot,nf))
rfir  : real interstitial function (in,real(ngtot,nf))
```

### DESCRIPTION:

Produces a 2D plot of the real functions contained in arrays `rfmt` and `rfir` on the parallelogram defined by the corner vertices in the global array `vclp2d`. See routine `rfarray`.

### REVISION HISTORY:

Created June 2003 (JKD)

---

## 7.115 plot3d (Source File: plot3d.f90)

### INTERFACE:

```
subroutine plot3d(fnum,nf,rfmt,rfir)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
fnum : plot file number (in,integer)
nf    : number of functions (in,integer)
rfmt  : real muffin-tin function (in,real(lmmaxvr,nrmtmax,natmtot,nf))
rfir  : real interstitial function (in,real(ngtot,nf))
```

### DESCRIPTION:

Produces a 3D plot of the real functions contained in arrays `rfmt` and `rfir` in the parallelepiped defined by the corner vertices in the global array `vclp3d`. See routine `rfarray`.

### REVISION HISTORY:

Created June 2003 (JKD)

Modified, October 2008 (F. Bultmark, F. Cricchio, L. Nordstrom)

---

## 7.116 `polynom` (Source File: `polynom.f90`)

### INTERFACE:

```
real(8) function polynom(m,np,xa,ya,c,x)
```

### INPUT/OUTPUT PARAMETERS:

```
  m : order of derivative (in,integer)
  np : number of points to fit (in,integer)
  xa : abscissa array (in,real(np))
  ya : ordinate array (in,real(np))
  c : work array (out,real(np))
  x : evaluation abscissa (in,real)
```

### DESCRIPTION:

Fits a polynomial of order  $n_p - 1$  to a set of  $n_p$  points. If  $m \geq 0$  the function returns the  $m$ th derivative of the polynomial at  $x$ , while for  $m < 0$  the integral of the polynomial from the first point in the array to  $x$  is returned.

### REVISION HISTORY:

Created October 2002 (JKD)

---

## 7.117 `potcoul` (Source File: `potcoul.f90`)

### INTERFACE:

```
subroutine potcoul
```

### USES:

```
use modmain
```

### DESCRIPTION:

Calculates the Coulomb potential of the real charge density stored in the global variables `rhomt` and `rhoir` by solving Poisson's equation. These variables are converted to complex representations and passed to the routine `zpotcoul`.

### REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.118 potks (Source File: potks.f90)

#### INTERFACE:

```
subroutine potks
```

#### *USES:*

```
use modmain
```

#### DESCRIPTION:

Computes the Kohn-Sham effective potential by adding together the Coulomb and exchange-correlation potentials. Also computes the effective magnetic field. See routines `potcoul` and `potxc`.

#### REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.119 potplot (Source File: potplot.f90)

#### INTERFACE:

```
subroutine potplot
```

#### *USES:*

```
use modmain
```

#### DESCRIPTION:

Outputs the exchange, correlation and Coulomb potentials, read in from `STATE.OUT`, for 1D, 2D or 3D plotting.

#### REVISION HISTORY:

Created June 2003 (JKD)

---

### 7.120 potxc (Source File: potxc.f90)

#### INTERFACE:

```
subroutine potxc
```

#### *USES:*

```
use modmain  
use modxcifc
```

## DESCRIPTION:

Computes the exchange-correlation potential and energy density. In the muffin-tin, the density is transformed from spherical harmonic coefficients  $\rho_{lm}$  to spherical coordinates  $(\theta, \phi)$  with a backward spherical harmonic transformation (SHT). Once calculated, the exchange-correlation potential and energy density are transformed with a forward SHT.

## REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.121 r3cross (Source File: r3cross.f90)

#### INTERFACE:

```
subroutine r3cross(x,y,z)
```

#### INPUT/OUTPUT PARAMETERS:

```
  x : input vector 1 (in,real(3))  
  y : input vector 2 (in,real(3))  
  z : output cross-product (out,real(3))
```

#### DESCRIPTION:

Returns the cross product of two real 3-vectors.

#### REVISION HISTORY:

Created September 2002 (JKD)

---

### 7.122 r3frac (Source File: r3frac.f90)

#### INTERFACE:

```
subroutine r3frac(eps,v)
```

#### INPUT/OUTPUT PARAMETERS:

```
  eps : zero component tolerance (in,real)  
  v    : input vector (inout,real(3))
```

#### DESCRIPTION:

Finds the fractional part of each component of a real 3-vector using the function  $\text{frac}(x) = x - \lfloor x \rfloor$ . A component is taken to be zero if it lies within the intervals  $[0, \epsilon)$  or  $(1 - \epsilon, 1]$ .

#### REVISION HISTORY:

Created January 2003 (JKD)  
Removed iv, September 2011 (JKD)

---

### 7.123 r3mdet (Source File: r3mdet.f90)

INTERFACE:

```
real(8) function r3mdet(a)
```

*INPUT/OUTPUT PARAMETERS:*

```
  a : input matrix (in,real(3,3))
```

DESCRIPTION:

Returns the determinant of a real  $3 \times 3$  matrix  $A$ .

REVISION HISTORY:

```
  Created May 2003 (JKD)
```

---

### 7.124 r3minv (Source File: r3minv.f90)

INTERFACE:

```
subroutine r3minv(a,b)
```

*INPUT/OUTPUT PARAMETERS:*

```
  a : input matrix (in,real(3,3))  
  b : output matrix (in,real(3,3))
```

DESCRIPTION:

Computes the inverse of a real  $3 \times 3$  matrix.

REVISION HISTORY:

```
  Created April 2003 (JKD)
```

---

### 7.125 r3mm (Source File: r3mm.f90)

INTERFACE:

```
subroutine r3mm(a,b,c)
```

*INPUT/OUTPUT PARAMETERS:*

```
  a : input matrix 1 (in,real(3,3))  
  b : input matrix 2 (in,real(3,3))  
  c : output matrix (out,real(3,3))
```

DESCRIPTION:

Multiplies two real  $3 \times 3$  matrices.

REVISION HISTORY:

Created April 2003 (JKD)

---

**7.126 r3mmt (Source File: r3mmt.f90)**

INTERFACE:

```
subroutine r3mmt(a,b,c)
```

*INPUT/OUTPUT PARAMETERS:*

```
  a : input matrix 1 (in,real(3,3))  
  b : input matrix 2 (in,real(3,3))  
  c : output matrix (out,real(3,3))
```

DESCRIPTION:

Multiplies a real matrix with the transpose of another.

REVISION HISTORY:

Created January 2003 (JKD)

---

**7.127 r3mtm (Source File: r3mtm.f90)**

INTERFACE:

```
subroutine r3mtm(a,b,c)
```

*INPUT/OUTPUT PARAMETERS:*

```
  a : input matrix 1 (in,real(3,3))  
  b : input matrix 2 (in,real(3,3))  
  c : output matrix (out,real(3,3))
```

DESCRIPTION:

Multiplies the transpose of one real  $3 \times 3$  matrix with another.

REVISION HISTORY:

Created January 2003 (JKD)

---

### 7.128 r3mtv (Source File: r3mtv.f90)

#### INTERFACE:

```
subroutine r3mtv(a,x,y)
```

#### *INPUT/OUTPUT PARAMETERS:*

```
  a : input matrix (in,real(3,3))  
  x : input vector (in,real(3))  
  y : output vector (out,real(3))
```

#### DESCRIPTION:

Multiplies the transpose of a real  $3 \times 3$  matrix with a vector.

#### REVISION HISTORY:

Created January 2003 (JKD)

---

### 7.129 r3mv (Source File: r3mv.f90)

#### INTERFACE:

```
subroutine r3mv(a,x,y)
```

#### *INPUT/OUTPUT PARAMETERS:*

```
  a : input matrix (in,real(3,3))  
  x : input vector (in,real(3))  
  y : output vector (out,real(3))
```

#### DESCRIPTION:

Multiplies a real  $3 \times 3$  matrix with a vector.

#### REVISION HISTORY:

Created January 2003 (JKD)

---

### 7.130 radnucl (Source File: radnucl.f90)

#### INTERFACE:

```
real(8) function radnucl(z)
```

#### *INPUT/OUTPUT PARAMETERS:*

```
  z : atomic number (in,real)
```

## DESCRIPTION:

Computes an approximate nuclear charge radius from the atomic number  $Z$ . The nuclear mass number is estimated using

$$A = 4.467 \times 10^{-3} Z^2 + 2.163Z - 1.168,$$

[D. Andrae in *Relativistic Electronic Structure Theory - Fundamentals* **11**, 203 (2002)], and the nuclear charge radius can be determined from

$$r = \left( r_0 + \frac{r_1}{A^{2/3}} + \frac{r_2}{A^{4/3}} \right) A^{1/3},$$

where  $r_0 = 0.9071$ ,  $r_1 = 1.105$  and  $r_2 = -0.548$  [I. Angeli, *Atomic Data and Nuclear Data Tables* **87**, 185 (2004)].

## REVISION HISTORY:

Created October 2011 (JKD)

---

### 7.131 rdiracdme (Source File: rdiracdme.f90)

#### INTERFACE:

```
subroutine rdiracdme(sol,m,kpa,e,nr,r,vr,nn,g0,g1,f0,f1)
```

#### INPUT/OUTPUT PARAMETERS:

```
sol : speed of light in atomic units (in,real)
m   : order of energy derivative (in,integer)
kpa : quantum number kappa (in,integer)
e   : energy (in,real)
nr  : number of radial mesh points (in,integer)
r   : radial mesh (in,real(nr))
vr  : potential on radial mesh (in,real(nr))
nn  : number of nodes (out,integer)
g0  : m th energy derivative of the major component multiplied by r
      (out,real(nr))
g1  : radial derivative of g0 (out,real(nr))
f0  : m th energy derivative of the minor component multiplied by r
      (out,real(nr))
f1  : radial derivative of f0 (out,real(nr))
```

#### DESCRIPTION:

Finds the solution to the  $m$ th energy derivative of the radial Dirac equation using the routine `rdiracint`.

#### REVISION HISTORY:

Created March 2003 (JKD)

---

### 7.132 rdirac (Source File: rdirac.f90)

#### INTERFACE:

```
subroutine rdirac(sol,n,l,k,nr,r,vr,eval,g0,f0)
```

#### INPUT/OUTPUT PARAMETERS:

```
sol  : speed of light in atomic units (in,real)
n    : principal quantum number (in,integer)
l    : quantum number l (in,integer)
k    : quantum number k (l or l+1) (in,integer)
nr   : number of radial mesh points (in,integer)
r    : radial mesh (in,real(nr))
vr   : potential on radial mesh (in,real(nr))
eval : eigenvalue without rest-mass energy (inout,real)
g0   : major component of the radial wavefunction (out,real(nr))
f0   : minor component of the radial wavefunction (out,real(nr))
```

#### DESCRIPTION:

Finds the solution to the radial Dirac equation for a given potential  $v(r)$  and quantum numbers  $n$ ,  $k$  and  $l$ . The method involves integrating the equation using the predictor-corrector method and adjusting  $E$  until the number of nodes in the wavefunction equals  $n - l - 1$ . The calling routine must provide an initial estimate for the eigenvalue. Note that the arrays `g0` and `f0` represent the radial functions multiplied by  $r$ .

#### REVISION HISTORY:

Created September 2002 (JKD)

---

### 7.133 rdiracint (Source File: rdiracint.f90)

#### INTERFACE:

```
subroutine rdiracint(sol,m,kpa,e,nr,r,vr,trsc,nn,g0p,f0p,g0,g1,f0,f1)
```

#### INPUT/OUTPUT PARAMETERS:

```
sol  : speed of light in atomic units (in,real)
m    : order of energy derivative (in,integer)
kpa  : quantum number kappa (in,integer)
e    : energy (in,real)
nr   : number of radial mesh points (in,integer)
r    : radial mesh (in,real(nr))
vr   : potential on radial mesh (in,real(nr))
trsc : .true. if the radial functions should be rescaled if any one of them
       exceeds an overflow value (in,logical)
```

nn : number of nodes (out,integer)  
 g0p : m-1 th energy derivative of the major component multiplied by r  
 (in,real(nr))  
 f0p : m-1 th energy derivative of the minor component multiplied by r  
 (in,real(nr))  
 g0 : m th energy derivative of the major component multiplied by r  
 (out,real(nr))  
 g1 : radial derivative of g0 (out,real(nr))  
 f0 : m th energy derivative of the minor component multiplied by r  
 (out,real(nr))  
 f1 : radial derivative of f0 (out,real(nr))

#### DESCRIPTION:

Integrates the  $m$ th energy derivative of the radial Dirac equation from  $r = 0$  outwards. This involves using the predictor-corrector method to solve the coupled first-order equations (in atomic units)

$$\begin{aligned} \left(\frac{d}{dr} + \frac{\kappa}{r}\right) G_{\kappa}^{(m)} &= \frac{1}{c} \{2E_0 + E - V\} F_{\kappa}^{(m)} + \frac{m}{c} F_{\kappa}^{(m-1)} \\ \left(\frac{d}{dr} - \frac{\kappa}{r}\right) F_{\kappa}^{(m)} &= -\frac{1}{c} \{E - V\} G_{\kappa}^{(m)} - \frac{m}{c} G_{\kappa}^{(m-1)}, \end{aligned}$$

where  $G_{\kappa}^{(m)} = r g_{\kappa}^{(m)}$  and  $F_{\kappa}^{(m)} = r f_{\kappa}^{(m)}$  are the  $m$ th energy derivatives of the major and minor components multiplied by  $r$ , respectively;  $V$  is the external potential;  $E_0$  is the electron rest energy;  $E$  is the eigen energy (excluding  $E_0$ ); and  $\kappa = l$  for  $j = l - \frac{1}{2}$  or  $\kappa = -(l + 1)$  for  $j = l + \frac{1}{2}$ . If  $m = 0$  then the arrays `g0p` and `f0p` are not referenced.

#### REVISION HISTORY:

Created September 2002 (JKD)  
 Polynomial order fixed to 3, September 2013 (JKD)

### 7.134 rdmdedc (Source File: rdmdedc.f90)

#### INTERFACE:

```
subroutine rdmdedc(dedc)
```

#### USES:

```
use modmain
use modrdm
```

#### INPUT/OUTPUT PARAMETERS:

dedc : energy derivative (out,complex(nstsv,nstsv,nkpt))

DESCRIPTION:

Calculates the derivative of the total energy w.r.t. the second-variational coefficients `evcsv`.

REVISION HISTORY:

Created 2008 (Sharma)

---

**7.135 rdmdedn (Source File: rdmdedn.f90)**

INTERFACE:

```
subroutine rdmdedn(dedn)
```

*USES:*

```
use modmain
use modrdm
```

*INPUT/OUTPUT PARAMETERS:*

```
dedn : free energy derivative (out,real(nstsv,nkpt))
```

DESCRIPTION:

Calculates the negative of the derivative of total free energy w.r.t. occupation numbers.

REVISION HISTORY:

Created 2008 (Sharma)

---

**7.136 rdmdexcdc (Source File: rdmdexcdc.f90)**

INTERFACE:

```
subroutine rdmdexcdc(dedc)
```

*USES:*

```
use modmain
use modrdm
```

*INPUT/OUTPUT PARAMETERS:*

```
dedc : energy derivative (inout,complex(nstsv,nstsv,nkpt))
```

DESCRIPTION:

Calculates the derivative of the exchange-correlation energy w.r.t. `evcsv` and adds the result to the total.

REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.137 rmdexcdn (Source File: rmdexcdn.f90)

#### INTERFACE:

```
subroutine rmdexcdn(dedn)
```

#### USES:

```
use modmain  
use modrdm
```

#### INPUT/OUTPUT PARAMETERS:

```
dedn : energy derivative (inout,real(nstsv,nkpt))
```

#### DESCRIPTION:

Calculates the derivative of the exchange-correlation energy w.r.t. occupation numbers and adds the result to the total.

#### REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.138 rdmdkdc (Source File: rdmdkdc.f90)

#### INTERFACE:

```
subroutine rdmdkdc
```

#### USES:

```
use modmain  
use modrdm
```

#### DESCRIPTION:

Calculates the derivative of kinetic energy w.r.t. the second-variational coefficients `evcsv`.

#### REVISION HISTORY:

Created October 2008 (Sharma)

---

### 7.139 rdmtdsdn (Source File: rdmtdsdn.f90)

#### INTERFACE:

```
subroutine rdmtdsdn(dedn)
```

#### USES:

```
use modmain  
use modrdm
```

#### INPUT/OUTPUT PARAMETERS:

```
dedn : energy derivative (inout,real(nstsv,nkpt))
```

#### DESCRIPTION:

Calculates the derivative of the entropic contribution to the free energy w.r.t. occupation numbers and adds it to the total.

#### REVISION HISTORY:

```
Created 2008 (Baldsiefen)
```

---

### 7.140 rdmenergy (Source File: rdmenergy.f90)

#### INTERFACE:

```
subroutine rdmenergy
```

#### USES:

```
use modmain  
use modrdm  
use modmpi  
use modtest
```

#### DESCRIPTION:

Calculates RDMFT total energy (free energy for finite temperatures).

#### REVISION HISTORY:

```
Created 2008 (Sharma)  
Updated for free energy 2009 (Baldsiefen)
```

---

### 7.141 rdmengyxc (Source File: rdmengyxc.f90)

#### INTERFACE:

```
subroutine rdmengyxc
```

#### USES:

```
use modmain  
use modrdm
```

#### DESCRIPTION:

Calculates RDMFT exchange-correlation energy.

#### REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.142 rdmentropy (Source File: rdmentropy.f90)

#### INTERFACE:

```
subroutine rdmentropy
```

#### USES:

```
use modmain  
use modrdm
```

#### DESCRIPTION:

Calculates RDMFT entropy  $S = -\sum_i n_i \log(n_i/n_{\max}) + (n_{\max} - n_i) \log(1 - n_i/n_{\max})$ , where  $n_{\max}$  is the maximum allowed occupancy (1 or 2).

#### REVISION HISTORY:

Created 2008 (Baldsiefen)

---

### 7.143 rdmeval (Source File: rdmeval.f90)

#### INTERFACE:

```
subroutine rdmeval
```

#### USES:

```
use modmain  
use modrdm
```

## DESCRIPTION:

RDMFT eigenvalues are determined by calculating the derivative of the total energy with respect to the occupation number at half the maximum occupancy ( $n_{\max}/2$ ).

## REVISION HISTORY:

Created 2009 (Sharma)

---

### 7.144 rdmft (Source File: rdmft.f90)

#### INTERFACE:

```
subroutine rdmft
```

#### *USES:*

```
use modmain  
use modrdm  
use modmpi
```

#### DESCRIPTION:

Main routine for one-body reduced density matrix functional theory (RDMFT).

#### REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.145 rdmminc (Source File: rdmminc.f90)

#### INTERFACE:

```
subroutine rdmminc
```

#### *USES:*

```
use modmain  
use modrdm  
use modmpi
```

#### DESCRIPTION:

Minimizes the total energy with respect to the second-variational coefficients `evcsv`. The steepest-descent algorithm is used.

#### REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.146 rdmminn (Source File: rdmminn.f90)

#### INTERFACE:

```
subroutine rdmminn
```

#### *USES:*

```
use modmain  
use modrdm  
use modmpi
```

#### DESCRIPTION:

Minimizes the total energy w.r.t. occupation numbers. The steepest-descent algorithm is used.

#### REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.147 rdmvaryc (Source File: rdmvaryc.f90)

#### INTERFACE:

```
subroutine rdmvaryc
```

#### *USES:*

```
use modmain  
use modrdm
```

#### DESCRIPTION:

Calculates new *evcsv* from old by using the derivatives of the total energy w.r.t. *evcsv*. A single step of steepest-descent is made.

#### REVISION HISTORY:

Created 2009 (Sharma)

---

### 7.148 rdmvaryn (Source File: rdmvaryn.f90)

#### INTERFACE:

```
subroutine rdmvaryn
```

#### *USES:*

```
use modmain
use modrdm
use modmpi
```

DESCRIPTION:

Calculates new occupation numbers from old by using the derivatives of the total energy:  
 $n_i^{\text{new}} = n_i^{\text{old}} - \tau \gamma_i$ , where  $\tau$  is chosen such that  $0 \leq n_i \leq n_{\text{max}}$  with

$$\gamma_i = \begin{cases} g_i(n_{\text{max}} - n_i) & g_i > 0 \\ g_i n_i & g_i \leq 0 \end{cases}$$

where  $g_i = \partial E / \partial n_i - \kappa$ , and  $\kappa$  is chosen such that  $\sum_i \gamma_i = 0$ .

REVISION HISTORY:

Created 2009 (JKD,Sharma)

---

### 7.149 rdmwritededn (Source File: rdmwritededn.f90)

INTERFACE:

```
subroutine rdmwritededn(dedn)
```

*USES:*

```
use modmain
use modrdm
```

*INPUT/OUTPUT PARAMETERS:*

dedn : derivative of energy (in,real(nstsv,nkpt))

DESCRIPTION:

Writes the derivative of total energy with respect to occupation numbers to file RDM\_DEDN.OUT.

REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.150 rdmwriteengy (Source File: rdmwriteengy.f90)

INTERFACE:

```
subroutine rdmwriteengy(fnum)
```

*USES:*

```
use modmain
use modrdm
```

*INPUT/OUTPUT PARAMETERS:*

fnum : file number for writing output (in,integer)

DESCRIPTION:

Writes all contributions to the total energy to file.

REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.151 readfermi (Source File: readfermi.f90)

INTERFACE:

```
subroutine readfermi
```

*USES:*

```
use modmain
```

DESCRIPTION:

Reads the Fermi energy from the file `EFERMI.OUT`.

REVISION HISTORY:

Created March 2005 (JKD)

---

### 7.152 readinput (Source File: readinput.f90)

INTERFACE:

```
subroutine readinput
```

*USES:*

```
use modmain
use modldapu
use modrdm
use modphonon
use modmpi
use modtest
use modrandom
use modscdft
use modpw
use modvars
```

## DESCRIPTION:

Reads in the input parameters from the file `elk.in`. Also sets default values for the input parameters.

## REVISION HISTORY:

Created September 2002 (JKD)

---

### 7.153 readstate (Source File: `readstate.f90`)

#### INTERFACE:

```
subroutine readstate
```

#### *USES:*

```
use modmain  
use modldapu
```

#### DESCRIPTION:

Reads in the charge density and other relevant variables from the file `STATE.OUT`. Checks for version and parameter compatibility.

#### REVISION HISTORY:

Created May 2003 (JKD)

---

### 7.154 reciplat (Source File: `reciplat.f90`)

#### INTERFACE:

```
subroutine reciplat
```

#### *USES:*

```
use modmain
```

#### DESCRIPTION:

Generates the reciprocal lattice vectors from the real-space lattice vectors

$$\mathbf{b}_1 = \frac{2\pi}{s}(\mathbf{a}_2 \times \mathbf{a}_3)$$
$$\mathbf{b}_2 = \frac{2\pi}{s}(\mathbf{a}_3 \times \mathbf{a}_1)$$
$$\mathbf{b}_3 = \frac{2\pi}{s}(\mathbf{a}_1 \times \mathbf{a}_2)$$

and finds the unit cell volume  $\Omega = |s|$ , where  $s = \mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)$ , and the Brillouin zone volume  $\Omega_{\text{BZ}} = (2\pi)^3/\Omega$ .

#### REVISION HISTORY:

Created September 2002 (JKD)

---

### 7.155 rfinp (Source File: rfinp.f90)

#### INTERFACE:

```
real(8) function rfinp(lrstp,rfmt1,rfmt2,rfir1,rfir2)
```

#### USES:

```
use modmain
```

#### INPUT/OUTPUT PARAMETERS:

```
lrstp : radial step length (in,integer)
rfmt1 : first function in real spherical harmonics for all muffin-tins
       (in,real(lmmaxvr,nrmtmax,natmtot))
rfmt2 : second function in real spherical harmonics for all muffin-tins
       (in,real(lmmaxvr,nrmtmax,natmtot))
rfir1 : first real interstitial function in real-space (in,real(ngtot))
rfir2 : second real interstitial function in real-space (in,real(ngtot))
```

#### DESCRIPTION:

Calculates the inner product of two real functions over the entire unit cell. The input muffin-tin functions should have angular momentum cut-off  $l_{\text{maxvr}}$ . In the interstitial region, the integrand is multiplied with the characteristic function,  $\tilde{\Theta}(\mathbf{r})$ , to remove the contribution from the muffin-tin. See routines `rfmtinp` and `gencfun`.

#### REVISION HISTORY:

Created July 2004 (JKD)

---

### 7.156 rfinterp (Source File: rfinterp.f90)

#### INTERFACE:

```
subroutine rfinterp(ni,xi,ldi,fi,no,xo,ldo,fo)
```

#### INPUT/OUTPUT PARAMETERS:

ni : number of input points (in,integer)  
xi : input abscissa array (in,real(ni))  
ldi : leading dimension (in,integer)  
fi : input data array (in,real(ldi,ni))  
no : number of output points (in,integer)  
xo : output abscissa array (in,real(ni))  
ldo : leading dimension (in,integer)  
fo : output interpolated function (out,real(ldo,no))

DESCRIPTION:

Given a function defined on a set of input points, this routine uses a clamped cubic spline to interpolate the function on a different set of points. See routine `spline`.

REVISION HISTORY:

Created January 2005 (JKD)

---

### 7.157 `rfmtctof` (Source File: `rfmtctof.f90`)

INTERFACE:

```
subroutine rfmtctof(rfmt)
```

*INPUT/OUTPUT PARAMETERS:*

```
rfmt : real muffin-tin function (in,real(lmmaxvr,nrmtmax,natmtot))
```

DESCRIPTION:

Converts a real muffin-tin function from a coarse to a fine radial mesh by using cubic spline interpolation. See `rfinterp` and `spline`.

REVISION HISTORY:

Created October 2003 (JKD)

---

### 7.158 `rfmtinp` (Source File: `rfmtinp.f90`)

INTERFACE:

```
real(8) function rfmtinp(lrstp,nr,nri,r,rfmt1,rfmt2)
```

*USES:*

```
use modmain
```

*INPUT/OUTPUT PARAMETERS:*

lrstp : radial step length (in,integer)  
 nr : number of radial mesh points (in,integer)  
 nri : number of radial mesh points on the inner part of the muffin-tin  
 (in,integer)  
 r : radial mesh (in,real(nr))  
 rfmt1 : first real function inside muffin-tin (in,real(lmmaxvr,nr))  
 rfmt2 : second real function inside muffin-tin (in,real(lmmaxvr,nr))

DESCRIPTION:

Calculates the inner product of two real functions in the muffin-tin. So given two real functions of the form

$$f(\mathbf{r}) = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l f_{lm}(r) R_{lm}(\hat{\mathbf{r}})$$

where  $R_{lm}$  are the real spherical harmonics, the function returns

$$I = \int \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l f_{lm}^1(r) f_{lm}^2(r) r^2 dr .$$

The radial integral is performed using a high accuracy cubic spline method. See routines `genrlm` and `fderiv`.

REVISION HISTORY:

Created November 2003 (Sharma)

---

## 7.159 rhocore (Source File: rhocore.f90)

INTERFACE:

subroutine rhocore

USES:

use modmain

DESCRIPTION:

Adds the core density and magnetisation to the muffin-tin functions. Also computes the amount of leakage of core charge from the muffin-tin spheres into the interstitial.

REVISION HISTORY:

Created April 2003 (JKD)

Fixed core moment direction, October 2012 (M. Meinert)

---

## 7.160 rhoinit (Source File: rhoinit.f90)

### INTERFACE:

```
subroutine rhoinit
```

### USES:

```
use modmain
```

### DESCRIPTION:

Initialises the crystal charge density. Inside the muffin-tins it is set to the spherical atomic density. In the interstitial region it is taken to be constant such that the total charge is correct. Requires that the atomic densities have already been calculated.

### REVISION HISTORY:

Created January 2003 (JKD)

---

## 7.161 rhomagk (Source File: rhomagk.f90)

### INTERFACE:

```
subroutine rhomagk(ik, evecfv, evecsv)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
ik      : k-point number (in, integer)
evecfv  : first-variational eigenvectors (in, complex(nmatmax, nstfv, nspnfv))
evecsv  : second-variational eigenvectors (in, complex(nstsv, nstsv))
```

### DESCRIPTION:

Generates the partial valence charge density from the eigenvectors at  $k$ -point  $ik$ . In the muffin-tin region, the wavefunction is obtained in terms of its  $(l, m)$ -components from both the APW and local-orbital functions. Using a backward spherical harmonic transform (SHT), the wavefunction is converted to real-space and the density obtained from its modulus squared. This density is then accumulated in the global variable `rhomt`. A similar process is used for the interstitial density in which the wavefunction in real-space is obtained from a Fourier transform of the sum of APW functions. The interstitial density is added to the global array `rhoir`. See routines `wavefmt`, `genshtmat` and `seceqn`.

### REVISION HISTORY:

Created April 2003 (JKD)  
Removed conversion to spherical harmonics, January 2009 (JKD)  
Partially de-phased the muffin-tin magnetisation for spin-spirals,  
February 2009 (FC, FB & LN)  
Optimisations, July 2010 (JKD)

---

### 7.162 rhomagsh (Source File: rhomagsh.f90)

#### INTERFACE:

```
subroutine rhomagsh
```

#### USES:

```
use modmain
```

#### DESCRIPTION:

Converts the muffin-tin density and magnetisation from spherical coordinates to a spherical harmonic expansion. See `rhomagk`.

#### REVISION HISTORY:

Created January 2009 (JKD)

---

### 7.163 rhonorm (Source File: rhonorm.f90)

#### INTERFACE:

```
subroutine rhonorm
```

#### USES:

```
use modmain
```

#### DESCRIPTION:

Loss of precision of the calculated total charge can result because the muffin-tin density is computed on a set of  $(\theta, \phi)$  points and then transformed to a spherical harmonic representation. This routine adds a constant to the density so that the total charge is correct. If the error in total charge exceeds a certain tolerance then a warning is issued.

#### REVISION HISTORY:

Created April 2003 (JKD)

Changed from rescaling to adding, September 2006 (JKD)

---

### 7.164 rhoplot (Source File: rhoplot.f90)

#### INTERFACE:

```
subroutine rhoplot
```

#### USES:

use modmain

DESCRIPTION:

Outputs the charge density, read in from STATE.OUT, for 1D, 2D or 3D plotting.

REVISION HISTORY:

Created June 2003 (JKD)

---

## 7.165 rlmrot (Source File: rlmrot.f90)

INTERFACE:

subroutine rlmrot(p,alpha,beta,gamma,lmax,ld,d)

INPUT/OUTPUT PARAMETERS:

p : if p=-1 then the rotation matrix is improper (in,integer)  
alpha : first Euler angle (in,real)  
beta : second Euler angle (in,real)  
gamma : third Euler angle (in,real)  
lmax : maximum angular momentum (in,integer)  
ld : leading dimension (in,integer)  
d : real spherical harmonic rotation matrix (out,real(ld,\*))

DESCRIPTION:

Returns the rotation matrix in the basis of real spherical harmonics given the three Euler angles,  $(\alpha, \beta, \gamma)$ , and the parity,  $p$ , of the rotation. The matrix is determined using the formula of V. V. Nechaev, [*J. Struct. Chem.* **35**, 115 (1994)], suitably modified for our definition of the real spherical harmonics ( $m_1 > 0$ ,  $m_2 > 0$ ):

$$\begin{aligned}\Delta_{00}^l &= d_{00}^l, \\ \Delta_{m_1 0}^l &= \sqrt{2} (-1)^{m_1} d_{0m_1}^l \cos(m_1 \alpha), \\ \Delta_{0m_2}^l &= \sqrt{2} (-1)^{m_2} d_{m_2 0}^l \cos(m_2 \gamma), \\ \Delta_{-m_1 0}^l &= -\sqrt{2} d_{0m_1}^l \sin(m_1 \alpha), \\ \Delta_{0-m_2}^l &= \sqrt{2} d_{m_2 0}^l \sin(m_2 \gamma), \\ \Delta_{m_1 m_2}^l &= (-1)^{m_1} (-1)^{m_2} \{ \cos(m_1 \alpha) \cos(m_2 \gamma) [d_A + d_B] - \sin(m_1 \alpha) \sin(m_2 \gamma) [d_A - d_B] \}, \\ \Delta_{m_1 - m_2}^l &= (-1)^{m_1} \{ \sin(m_1 \alpha) \cos(m_2 \gamma) [d_A - d_B] + \cos(m_1 \alpha) \sin(m_2 \gamma) [d_A + d_B] \}, \\ \Delta_{-m_1 m_2}^l &= -(-1)^{m_2} \{ \sin(m_1 \alpha) \cos(m_2 \gamma) [d_A + d_B] + \cos(m_1 \alpha) \sin(m_2 \gamma) [d_A - d_B] \}, \\ \Delta_{-m_1 - m_2}^l &= \cos(m_1 \alpha) \cos(m_2 \gamma) [d_A - d_B] - \sin(m_1 \alpha) \sin(m_2 \gamma) [d_A + d_B],\end{aligned}$$

where  $d_A \equiv d_{-m_1 - m_2}^l$ ,  $d_B \equiv (-1)^{m_1} d_{m_1 - m_2}^l$  and  $d$  is the rotation matrix about the  $y$ -axis for complex spherical harmonics. See the routines `genrlm`, `euler` and `ylmroty`.

REVISION HISTORY:

Created December 2008 (JKD)

---

## 7.166 rotaxang (Source File: rotaxang.f90)

### INTERFACE:

```
subroutine rotaxang(eps,rot,det,v,th)
```

### INPUT/OUTPUT PARAMETERS:

```
eps : zero vector tolerance (in,real)
rot : rotation matrix (in,real(3,3))
det : matrix determinant (out,real)
v   : normalised axis vector (out,real(3))
th  : rotation angle (out,real)
```

### DESCRIPTION:

Given a rotation matrix

$$R(\hat{\mathbf{v}}, \theta) = \begin{pmatrix} \cos \theta + x^2(1 - \cos \theta) & xy(1 - \cos \theta) + z \sin \theta & xz(1 - \cos \theta) - y \sin \theta \\ xy(1 - \cos \theta) - z \sin \theta & \cos \theta + y^2(1 - \cos \theta) & yz(1 - \cos \theta) + x \sin \theta \\ xz(1 - \cos \theta) + y \sin \theta & yz(1 - \cos \theta) - x \sin \theta & \cos \theta + z^2(1 - \cos \theta) \end{pmatrix},$$

this routine determines the axis of rotation  $\hat{\mathbf{v}}$  and the angle of rotation  $\theta$ . If  $R$  corresponds to an improper rotation then only the proper part is used and **det** is set to  $-1$ .

### REVISION HISTORY:

Created December 2006 (JKD)

---

## 7.167 rotrflm (Source File: rotrflm.f90)

### INTERFACE:

```
subroutine rotrflm(rot,lmax,n,ld,rflm1,rflm2)
```

### INPUT/OUTPUT PARAMETERS:

```
rot   : rotation matrix (in,real(3,3))
lmax  : maximum angular momentum (in,integer)
n     : number of functions to rotate (in,integer)
ld    : leading dimension (in,integer)
rflm1 : coefficients of the real spherical harmonic expansion for each
       : function (in,real(ld,n))
rflm2 : coefficients of rotated functions (out,complex(ld,n))
```

### DESCRIPTION:

Rotates a set of real functions

$$f_i(\mathbf{r}) = \sum_{lm} f_{lm}^i R_{lm}(\hat{\mathbf{r}})$$

for all  $i$ , given the coefficients  $f_{lm}^i$  and a rotation matrix  $R$ . This is done by first the computing the Euler angles  $(\alpha, \beta, \gamma)$  of  $R^{-1}$  (see routine `euler`) and then applying the spherical harmonic rotation matrix generated by the routine `rlmrot`.

#### REVISION HISTORY:

Created December 2008 (JKD)

---

### 7.168 rotzflm (Source File: rotzflm.f90)

#### INTERFACE:

```
subroutine rotzflm(rot,lmin,lmax,n,ld,zflm1,zflm2)
```

#### INPUT/OUTPUT PARAMETERS:

```
rot   : rotation matrix (in,real(3,3))
lmin  : minimum angular momentum (in,integer)
lmax  : maximum angular momentum (in,integer)
n     : number of functions to rotate (in,integer)
ld    : leading dimension (in,integer)
zflm1 : coefficients of the complex spherical harmonic expansion for each
        function (in,complex(ld,n))
zflm2 : coefficients of rotated functions (out,complex(ld,n))
```

#### DESCRIPTION:

Rotates a set of complex functions

$$f_i(\mathbf{r}) = \sum_{lm} f_{lm}^i Y_{lm}(\hat{\mathbf{r}})$$

for all  $i$ , given the coefficients  $f_{lm}^i$  and a rotation matrix  $R$ . This is done by first the computing the Euler angles  $(\alpha, \beta, \gamma)$  of  $R^{-1}$  (see routine `euler`) and then applying the spherical harmonic rotation matrix generated by the routine `ylmrot`.

#### REVISION HISTORY:

Created April 2003 (JKD)  
 Modified, December 2008 (JKD)

---

### 7.169 rschrodapp (Source File: rschrodapp.f90)

#### INTERFACE:

```
subroutine rschrodapp(sol,l,nr,r,vr,p0,q0,q1,hp0)
```

#### INPUT/OUTPUT PARAMETERS:

sol : speed of light in atomic units (in,real)  
 l : angular momentum quantum number (in,integer)  
 nr : number of radial mesh points (in,integer)  
 r : radial mesh (in,real(nr))  
 vr : potential on radial mesh (in,real(nr))  
 p0 : m th energy derivative of P (in,real(nr))  
 q0 : m th energy derivative of Q (in,real(nr))  
 q1 : radial derivative of q0 (in,real(nr))  
 hp0 : H applied to P (out,real(nr))

DESCRIPTION:

Applies the scalar relativistic radial Hamiltonian,  $H$ , to a radial wavefunction,  $P_l$ . This is an approximation since we assume  $P_l$  is a scalar wavefunction, normalisable to unity. A Hamiltonian which satisfies  $HP_l = EP_l$  is given implicitly by

$$HP_l = \left[ \frac{l(l+1)}{2Mr^2} + V \right] P_l - \frac{1}{r} Q_l - \frac{d}{dr} Q_l,$$

where  $V$  is the external potential,  $M = 1 - V/2c^2$  and  $Q_l$  is obtained from integrating the coupled scalar relativistic equations.

REVISION HISTORY:

Created October 2003 (JKD)

**7.170 rschroddme (Source File: rschroddme.f90)**

INTERFACE:

subroutine rschroddme(sol,m,l,k,e,nr,r,vr,nn,p0,p1,q0,q1)

INPUT/OUTPUT PARAMETERS:

sol : speed of light in atomic units (in,real)  
 m : order of energy derivative (in,integer)  
 l : angular momentum quantum number (in,integer)  
 k : quantum number k, zero if Dirac eqn. is not to be used (in,integer)  
 e : energy (in,real)  
 nr : number of radial mesh points (in,integer)  
 r : radial mesh (in,real(nr))  
 vr : potential on radial mesh (in,real(nr))  
 nn : number of nodes (out,integer)  
 p0 : m th energy derivative of P (out,real(nr))  
 p1 : radial derivative of p0 (out,real(nr))  
 q0 : m th energy derivative of Q (out,real(nr))  
 q1 : radial derivative of q0 (out,real(nr))

## DESCRIPTION:

Finds the solution to the  $m$ th energy derivative of the scalar relativistic radial Schrödinger equation using the routine `rschrodint`.

## REVISION HISTORY:

Created June 2003 (JKD)

---

## 7.171 `rschrodint` (Source File: `rschrodint.f90`)

### INTERFACE:

```
subroutine rschrodint(sol,m,l,e,nr,r,vr,nn,p0p,p0,p1,q0,q1)
```

### INPUT/OUTPUT PARAMETERS:

```
sol : speed of light in atomic units (in,real)
m   : order of energy derivative (in,integer)
l   : angular momentum quantum number (in,integer)
e   : energy (in,real)
nr  : number of radial mesh points (in,integer)
r   : radial mesh (in,real(nr))
vr  : potential on radial mesh (in,real(nr))
nn  : number of nodes (out,integer)
p0p : m-1 th energy derivative of P (in,real(nr))
p0  : m th energy derivative of P (out,real(nr))
p1  : radial derivative of p0 (out,real(nr))
q0  : m th energy derivative of Q (out,real(nr))
q1  : radial derivative of q0 (out,real(nr))
```

### DESCRIPTION:

Integrates the  $m$ th energy derivative of the scalar relativistic radial Schrödinger equation from  $r = 0$  outwards. This involves using the predictor-corrector method to solve the coupled first-order equations (in atomic units)

$$\begin{aligned}\frac{d}{dr}P_l^{(m)} &= 2MQ_l^{(m)} + \frac{1}{r}P_l^{(m)} \\ \frac{d}{dr}Q_l^{(m)} &= -\frac{1}{r}Q_l^{(m)} + \left[ \frac{l(l+1)}{2Mr^2} + (V - E) \right] P_l^{(m)} - mP_l^{(m-1)},\end{aligned}$$

where  $V$  is the external potential,  $E$  is the eigenenergy and  $M = 1 - V/2c^2$ . Following the convention of Koelling and Harmon, *J. Phys. C: Solid State Phys.* **10**, 3107 (1977), the functions  $P_l$  and  $Q_l$  are defined by

$$\begin{aligned}P_l &= rg_l \\ Q_l &= \frac{r}{2M} \frac{dg_l}{dr},\end{aligned}$$

where  $g_l$  is the major component of the Dirac equation (see the routine `rdiracint`). Note that in order to make the equations linear in energy, the full definition  $M = 1 + (E - V)/2c^2$  is not used. If  $m = 0$  then the array `p0p` is not referenced.

#### REVISION HISTORY:

Created October 2003 (JKD)

---

### 7.172 `rtozflm` (Source File: `rtozflm.f90`)

#### INTERFACE:

```
subroutine rtozflm(lmax,rflm,zflm)
```

#### INPUT/OUTPUT PARAMETERS:

`lmax` : maximum angular momentum (in,integer)  
`rflm` : coefficients of real spherical harmonic expansion  
(in,real((lmax+1)\*\*2))  
`zflm` : coefficients of complex spherical harmonic expansion  
(out,complex((lmax+1)\*\*2))

#### DESCRIPTION:

Converts a real function,  $r_{lm}$ , expanded in terms of real spherical harmonics into a complex spherical harmonic expansion,  $z_{lm}$ :

$$z_{lm} = \begin{cases} \frac{1}{\sqrt{2}}(r_{lm} + i(-1)^m r_{l-m}) & m > 0 \\ \frac{1}{\sqrt{2}}((-1)^m r_{l-m} - i r_{lm}) & m < 0 \\ r_{lm} & m = 0 \end{cases} .$$

See routine `genrlm`.

#### REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.173 `rvfcross` (Source File: `rvfcross.f90`)

#### INTERFACE:

```
subroutine rvfcross(rvfmt1,rvfmt2,rvfir1,rvfir2,rvfmt3,rvfir3)
```

#### USES:

```
use modmain
```

*INPUT/OUTPUT PARAMETERS:*

rvfmt1 : first input muffin-tin field (in,real(lmmaxvr,nrmtmax,natmtot,3))  
rvfmt2 : second input muffin-tin field (in,real(lmmaxvr,nrmtmax,natmtot,3))  
rvfir1 : first input interstitial field (in,real(ngtot,3))  
rvfir2 : second input interstitial field (in,real(ngtot,3))  
rvfmt3 : output muffin-tin field (out,real(lmmaxvr,nrmtmax,natmtot,3))  
rvfir3 : output interstitial field (out,real(ngtot,3))

DESCRIPTION:

Given two real vector fields,  $\mathbf{f}_1$  and  $\mathbf{f}_2$ , defined over the entire unit cell, this routine computes the local cross product

$$\mathbf{f}_3(\mathbf{r}) \equiv \mathbf{f}_1(\mathbf{r}) \times \mathbf{f}_2(\mathbf{r}).$$

REVISION HISTORY:

Created February 2007 (JKD)

---

**7.174 sbesseldm (Source File: sbesseldm.f90)**

INTERFACE:

subroutine sbesseldm(m,lmax,x,djl)

*INPUT/OUTPUT PARAMETERS:*

m : order of derivative (in,integer)  
lmax : maximum order of Bessel function (in,integer)  
x : real argument (in,real)  
djl : array of returned values (out,real(0:lmax))

DESCRIPTION:

Computes the  $m$ th derivative of the spherical Bessel function of the first kind,  $j_l(x)$ , for argument  $x$  and  $l = 0, 1, \dots, l_{\max}$ . For  $x \geq 1$  this is done by repeatedly using the relations

$$\frac{d}{dx} j_l(x) = \frac{l}{x} j_l(x) - j_{l+1}(x)$$
$$j_{l+1}(x) = \frac{2l+1}{x} j_l(x) - j_{l-1}(x).$$

While for  $x < 1$  the series expansion of the Bessel function is used

$$\frac{d^m}{dx^m} j_l(x) = \sum_{i=0}^{\infty} \frac{(2i+l)!}{(-2)^i i! (2i+l-m)! (2i+2l+1)!!} x^{2i+l-m}.$$

This procedure is numerically stable and accurate to near machine precision for  $l \leq 30$  and  $m \leq 6$ .

REVISION HISTORY:

Created March 2003 (JKD)  
Modified to return an array of values, October 2004 (JKD)

---

## 7.175 sbessel (Source File: sbessel.f90)

### INTERFACE:

```
subroutine sbessel(lmax,x,jl)
```

### INPUT/OUTPUT PARAMETERS:

```
lmax : maximum order of Bessel function (in,integer)  
x    : real argument (in,real)  
jl   : array of returned values (out,real(0:lmax))
```

### DESCRIPTION:

Computes the spherical Bessel functions of the first kind,  $j_l(x)$ , for argument  $x$  and  $l = 0, 1, \dots, l_{\max}$ . The recursion relation

$$j_{l+1}(x) = \frac{2l+1}{x}j_l(x) - j_{l-1}(x)$$

is used either downwards for  $x < l$  or upwards for  $x \geq l$ . For  $x \ll 1$  the asymptotic form is used

$$j_l(x) \approx \frac{x^l}{(2l+1)!!}.$$

This procedure is numerically stable and accurate to near machine precision for  $l \leq 50$ .

### REVISION HISTORY:

```
Created January 2003 (JKD)  
Modified to return an array of values, October 2004 (JKD)  
Improved stability, August 2006 (JKD)
```

---

## 7.176 sdelta (Source File: sdelta.f90)

### INTERFACE:

```
real(8) function sdelta(stype,x)
```

### INPUT/OUTPUT PARAMETERS:

```
stype : smearing type (in,integer)  
x     : real argument (in,real)
```

### DESCRIPTION:

Returns a normalised smooth approximation to the Dirac delta function. These functions are defined such that

$$\int \tilde{\delta}(x)dx = 1.$$

The effective width,  $w$ , of the delta function may be varied by using the normalising transformation

$$\tilde{\delta}_w(x) \equiv \frac{\tilde{\delta}(x/w)}{w}.$$

Currently implemented are:

- 0. Gaussian
- 1. Methfessel-Paxton order 1
- 2. Methfessel-Paxton order 2
- 3. Fermi-Dirac
- 4. Square-wave impulse
- 5. Lorentzian

See routines `stheta`, `sdelta_mp`, `sdelta_fd` and `sdelta_sq`.

REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.177 getsdata (Source File: `sdelta.f90`)

INTERFACE:

```
subroutine getsdata(stype,sdescr)
```

*INPUT/OUTPUT PARAMETERS:*

```
  stype : smearing type (in,integer)
  sdescr : smearing scheme description (out,character(256))
```

DESCRIPTION:

Returns a description of the smearing scheme as string `sdescr` up to 256 characters long.

REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.178 sdelta\_fd (Source File: `sdelta_fd.f90`)

INTERFACE:

```
real(8) function sdelta_fd(x)
```

*INPUT/OUTPUT PARAMETERS:*

```
  x : real argument (in,real)
```

DESCRIPTION:

Returns the Fermi-Dirac approximation to the Dirac delta function

$$\tilde{\delta}(x) = \frac{e^{-x}}{(1 + e^{-x})^2}.$$

REVISION HISTORY:

Created April 2003 (JKD)

---

## 7.179 `sdelta_mp` (Source File: `sdelta_mp.f90`)

INTERFACE:

```
real(8) function sdelta_mp(n,x)
```

INPUT/OUTPUT PARAMETERS:

```
  n : order (in,integer)
  x : real argument (in,real)
```

DESCRIPTION:

Returns the smooth approximation to the Dirac delta function of order  $N$  given by Methfessel and Paxton, *Phys. Rev. B* **40**, 3616 (1989),

$$\tilde{\delta}(x) = \sum_{i=0}^N \frac{(-1)^i}{i!4^i\sqrt{\pi}} H_{2i}(x) e^{-x^2},$$

where  $H_j$  is the  $j$ th-order Hermite polynomial. This function has the property

$$\int_{-\infty}^{\infty} \tilde{\delta}(x) P(x) dx = P(0),$$

where  $P(x)$  is any polynomial of degree  $2N + 1$  or less. The case  $N = 0$  corresponds to Gaussian smearing. This procedure is numerically stable and accurate to near machine precision for  $N \leq 10$ .

REVISION HISTORY:

Created April 2003 (JKD)

---

## 7.180 `sdelta_sq` (Source File: `sdelta_sq.f90`)

INTERFACE:

```
real(8) function sdelta_sq(x)
```

INPUT/OUTPUT PARAMETERS:

```
  x : real argument (in,real)
```

DESCRIPTION:

Returns the square-wave pulse approximation to the Dirac delta function

$$\tilde{\delta}(x) = \begin{cases} 1 & |x| \leq 1/2 \\ 0 & |x| > 1/2 \end{cases}$$

REVISION HISTORY:

Created July 2008 (JKD)

---

### 7.181 seceqn (Source File: seceqn.f90)

subroutine seceqn(ik,evalfv,evecfv,evecsv) *USES:*

```
use modmain
```

*INPUT/OUTPUT PARAMETERS:*

```
ik      : k-point number (in,integer)
evalfv  : first-variational eigenvalues (out,real(nstfv))
evecfv  : first-variational eigenvectors (out,complex(nmatmax,nstfv))
evecsv  : second-variational eigenvectors (out,complex(nstsv,nstsv))
```

DESCRIPTION:

Solves the first- and second-variational secular equations. See routines `match`, `seceqnfv`, `seceqnss` and `seceqnsv`.

REVISION HISTORY:

Created March 2004 (JKD)

---

### 7.182 seceqnfv (Source File: seceqnfv.f90)

INTERFACE:

```
subroutine seceqnfv(nmatp,ngp,igpig,vpc,vgpc,apwalm,evalfv,evecfv)
```

*USES:*

```
use modmain
```

*INPUT/OUTPUT PARAMETERS:*

```
nmatp  : order of overlap and Hamiltonian matrices (in,integer)
ngp    : number of G+k-vectors for augmented plane waves (in,integer)
igpig  : index from G+k-vectors to G-vectors (in,integer(ngkmax))
vpc    : k-vector in Cartesian coordinates (in,real(3))
vgpc   : G+k-vectors in Cartesian coordinates (in,real(3,ngkmax))
apwalm : APW matching coefficients
        (in,complex(ngkmax,apwordmax,lmmxapw,natmtot))
evalfv  : first-variational eigenvalues (out,real(nstfv))
evecfv  : first-variational eigenvectors (out,complex(nmatmax,nstfv))
```

DESCRIPTION:

Solves the secular equation,

$$(H - \epsilon O)b = 0,$$

for the all the first-variational states of the input  $k$ -point.

REVISION HISTORY:

Created March 2004 (JKD)

---

## 7.183 seceqnfvr (Source File: seceqnfvr.f90)

### INTERFACE:

```
subroutine seceqnfvr(nmatp,ngp,vpc,h,o,evalfv,vecfv)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
nmatp : order of overlap and Hamiltonian matrices (in,integer)
ngp   : number of G+k-vectors for augmented plane waves (in,integer)
vpc   : k-vector in Cartesian coordinates (in,real(3))
h,o   : Hamiltonian and overlap matrices in packed or upper triangular
       : form (in,complex(*))
evalfv : first-variational eigenvalues (out,real(nstfv))
vecfv  : first-variational eigenvectors (out,complex(nmatmax,nstfv))
```

### DESCRIPTION:

This routine solves the first-variational secular equation for the special case when inversion symmetry is present. In this case the Hamiltonian and overlap matrices can be made real by using appropriate linear combinations of the local-orbitals for atoms related by inversion symmetry. These are derived from the effect of parity and complex conjugation on the spherical harmonics:  $PY_{lm} = (-1)^l Y_{lm}$  and  $(Y_{lm})^* = (-1)^m Y_{l-m}$ .

### REVISION HISTORY:

Created May 2011 (JKD)

---

## 7.184 sfacmag (Source File: sfacmag.f90)

### INTERFACE:

```
subroutine sfacmag
```

### USES:

```
use modmain
use modpw
```

### DESCRIPTION:

Outputs magnetic structure factors, i.e. the Fourier transform coefficients of each component  $j$  of magnetization  $\mathbf{m}(\mathbf{r})$ ,

$$F_j(\mathbf{H}) = \int_{\Omega} d^3r m_j(\mathbf{r}) e^{i\mathbf{H}\cdot\mathbf{r}},$$

to the files SFACMAG\_j.OUT. The lattice coordinates  $(h,k,l)$  of  $\mathbf{H}$ -vectors in this file are transformed by the matrix `vhmat`. See also routines `zftrf` and `genhvec`.

### REVISION HISTORY:

Created July 2010 (Alexey I. Baranov)

Added multiplicity of the H-vectors, Oct. 2010 (Alexey I. Baranov)

---

### 7.185 sfacrho (Source File: sfacrho.f90)

INTERFACE:

```
subroutine sfacrho
```

*USES:*

```
use modmain
```

```
use modpw
```

DESCRIPTION:

Outputs X-ray structure factors, i.e. the Fourier transform coefficients of the total electron density

$$F(\mathbf{H}) = \int_{\Omega} d^3r \rho(\mathbf{r}) e^{i\mathbf{H}\cdot\mathbf{r}},$$

to the file SFACRHO.OUT. The lattice coordinates  $(h, k, l)$  of the  $\mathbf{H}$ -vectors in this file are transformed by the matrix `vhmat`. If an energy window is set using the variable `wsfac`, then only those states within the window are used to compute the density. See also routines `zftrf` and `genhvec`.

REVISION HISTORY:

Created July 2010 (Alexey I. Baranov)

Added multiplicity of the H-vectors, Oct. 2010 (Alexey I. Baranov)

---

### 7.186 sortidx (Source File: sortidx.f90)

INTERFACE:

```
subroutine sortidx(n,a,idx)
```

*INPUT/OUTPUT PARAMETERS:*

`n` : number of elements in array (in, integer)

`a` : real array (in, real(n))

`idx` : permutation index (out, integer(n))

DESCRIPTION:

Finds the permutation index `idx` which sorts the real array `a` into ascending order. No sorting of the array `a` itself is performed. Uses the heapsort algorithm.

REVISION HISTORY:

Created October 2002 (JKD)

Included tolerance `eps`, April 2006 (JKD)

---

## 7.187 sphcover (Source File: sphcover.f90)

INTERFACE:

```
subroutine sphcover(n,tp)
```

*INPUT/OUTPUT PARAMETERS:*

```
  n : number of required points (in,integer)  
  tp : (theta, phi) coordinates (out,real(2,n))
```

DESCRIPTION:

Produces a set of  $N$  points which cover the unit sphere nearly optimally. The points in spherical  $(\theta, \phi)$  coordinates are generated using the explicit ‘golden section’ formula:

$$\begin{aligned}\theta_k &= \arccos \left[ 1 - \left( k - \frac{1}{2} \right) \delta z \right] \\ \phi_k &= (k - 1) \delta \phi,\end{aligned}$$

where  $\delta z = 2/n$  and  $\delta \phi = \pi(1 - \sqrt{5})$ .

REVISION HISTORY:

```
  Created April 2008 (JKD)  
  Improved covering, October 2009 (JKD)
```

---

## 7.188 sphcrd (Source File: sphcrd.f90)

INTERFACE:

```
subroutine sphcrd(v,r,tp)
```

*INPUT/OUTPUT PARAMETERS:*

```
  v : input vector (in,real(3))  
  r : length of v (out,real)  
  tp : (theta, phi) coordinates (out,real(2))
```

DESCRIPTION:

Returns the spherical coordinates  $(r, \theta, \phi)$  of a vector

$$\mathbf{v} = (r \sin(\theta) \cos(\phi), r \sin(\theta) \sin(\phi), r \cos(\theta)).$$

REVISION HISTORY:

```
  Created October 2002 (JKD)
```

---

## 7.189 spline (Source File: spline.f90)

### INTERFACE:

```
subroutine spline(n,x,ld,f,cf)
```

### INPUT/OUTPUT PARAMETERS:

```
  n : number of points (in,integer)
  x : abscissa array (in,real(n))
  ld : leading dimension (in,integer)
  f : input data array (in,real(ld,n))
  cf : cubic spline coefficients (out,real(3,n))
```

### DESCRIPTION:

Calculates the coefficients of a cubic spline fitted to input data. In other words, given a set of data points  $f_i$  defined at  $x_i$ , where  $i = 1 \dots n$ , the coefficients  $c_j^i$  are determined such that

$$y_i(x) = f_i + c_1^i(x - x_i) + c_2^i(x - x_i)^2 + c_3^i(x - x_i)^3,$$

is the interpolating function for  $x \in [x_i, x_{i+1})$ . The coefficients are determined piecewise by fitting a cubic polynomial to adjacent points.

### REVISION HISTORY:

Created November 2011 (JKD)

---

## 7.190 stheta (Source File: stheta.f90)

### INTERFACE:

```
real(8) function stheta(stype,x)
```

### INPUT/OUTPUT PARAMETERS:

```
  stype : smearing type (in,integer)
  x      : real argument (in,real)
```

### DESCRIPTION:

Returns the Heaviside step function corresponding to the smooth approximation to the Dirac delta function:

$$\tilde{\Theta}(x) = \int_{-\infty}^x dt \tilde{\delta}(t).$$

See function `sdelta` for details.

### REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.191 stheta\_fd (Source File: stheta\_fd.f90)

INTERFACE:

```
real(8) function stheta_fd(x)
```

INPUT/OUTPUT PARAMETERS:

```
  x : real argument (in,real)
```

DESCRIPTION:

Returns the Fermi-Dirac approximation to the Heaviside step function

$$\tilde{\Theta}(x) = \frac{1}{1 + e^{-x}}.$$

REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.192 stheta\_mp (Source File: stheta\_mp.f90)

INTERFACE:

```
real(8) function stheta_mp(n,x)
```

INPUT/OUTPUT PARAMETERS:

```
  n : order (in,integer)
  x : real argument (in,real)
```

DESCRIPTION:

Returns the smooth approximation to the Heaviside step function of order  $N$  given by Methfessel and Paxton, *Phys. Rev. B* **40**, 3616 (1989),

$$\tilde{\Theta}(x) = 1 - S_N(x)$$

where

$$S_N(x) = S_0(x) + \sum_{i=1}^N \frac{(-1)^i}{i!4^i\sqrt{\pi}} H_{2i-1}(x)e^{-x^2},$$
$$S_0(x) = \frac{1}{2}(1 - \operatorname{erf}(x))$$

and  $H_j$  is the  $j$ th-order Hermite polynomial. This procedure is numerically stable and accurate to near machine precision for  $N \leq 10$ .

REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.193 stheta\_sq (Source File: stheta\_sq.f90)

INTERFACE:

```
real(8) function stheta_sq(x)
```

INPUT/OUTPUT PARAMETERS:

```
x : real argument (in,real)
```

DESCRIPTION:

Returns the Heaviside step function corresponding to the square-wave pulse approximation to the Dirac delta function

$$\tilde{\Theta}(x) = \begin{cases} 0 & x \leq -1/2 \\ x + 1/2 & -1/2 < x < 1/2 \\ 1 & x \geq 1/2 \end{cases}$$

REVISION HISTORY:

Created July 2008 (JKD)

---

### 7.194 sumrule (Source File: sumrule.f90)

INTERFACE:

```
subroutine sumrule(dynq)
```

INPUT/OUTPUT PARAMETERS:

```
dynq : dynamical matrices on q-point set (in,real(3*natmtot,3*natmtot,nqpt))
```

DESCRIPTION:

Applies the same correction to all the dynamical matrices such that the matrix for  $\mathbf{q} = 0$  satisfies the acoustic sum rule. In other words, the matrices are updated with

$$D_{ij}^{\mathbf{q}} \rightarrow D_{ij}^{\mathbf{q}} - \sum_{k=1}^3 \omega_k^0 v_{k;i}^0 v_{k;j}^0$$

for all  $\mathbf{q}$ , where  $\omega_k^0$  is the  $k$ th eigenvalue of the  $\mathbf{q} = 0$  dynamical matrix and  $v_{k;i}^0$  the  $i$ th component of its eigenvector. The eigenvalues are assumed to be arranged in ascending order. This ensures that the  $\mathbf{q} = 0$  dynamical matrix has 3 zero eigenvalues, which the uncorrected matrix may not have due to the finite exchange-correlation grid.

REVISION HISTORY:

Created May 2005 (JKD)

---

## 7.195 symrf (Source File: symrf.f90)

### INTERFACE:

```
subroutine symrf(lrstp,rfmt,rfir)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
lrstp : radial step length (in,integer)  
rfmt  : real muffin-tin function (inout,real(lmmaxvr,nrmtmax,natmtot))  
rfir  : real interstitial function (inout,real(ngtot))
```

### DESCRIPTION:

Symmetrises a real scalar function defined over the entire unit cell using the full set of crystal symmetries. In the muffin-tin of a particular atom the spherical harmonic coefficients of every equivalent atom are rotated and averaged. The interstitial part of the function is first Fourier transformed to  $G$ -space, and then averaged over each symmetry by rotating the Fourier coefficients and multiplying them by a phase factor corresponding to the symmetry translation.

### REVISION HISTORY:

Created May 2007 (JKD)

---

## 7.196 symrfir (Source File: symrfir.f90)

### INTERFACE:

```
subroutine symrfir(ngv,rfir)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
ngv  : number of G-vectors to be used for the Fourier space rotation  
      (in,integer)  
rfir : real interstitial function (inout,real(ngtot))
```

### DESCRIPTION:

Symmetrises a real scalar interstitial function. The function is first Fourier transformed to  $G$ -space, and then averaged over each symmetry by rotating the Fourier coefficients and multiplying them by a phase factor corresponding to the symmetry translation.

### REVISION HISTORY:

Created July 2007 (JKD)

---

## 7.197 symrvf (Source File: symrvf.f90)

INTERFACE:

```
subroutine symrvf(lrstp,rvfmt,rvfir)
```

*USES:*

```
use modmain
```

*INPUT/OUTPUT PARAMETERS:*

```
lrstp : radial step length (in,integer)
rvfmt : real muffin-tin vector field
       (in,real(lmmaxvr,nrmtmax,natmtot,ndmag))
rvfir : real interstitial vector field (in,real(ngtot,ndmag))
```

DESCRIPTION:

Symmetrises a vector field defined over the entire unit cell using the full set of crystal symmetries. If a particular symmetry involves rotating atom 1 into atom 2, then the spatial and spin rotations of that symmetry are applied to the vector field in atom 2 (expressed in spherical harmonic coefficients), which is then added to the field in atom 1. This is repeated for all symmetry operations. The fully symmetrised field in atom 1 is then rotated and copied to atom 2. Symmetrisation of the interstitial part of the field is performed by `symrvfir`. See also `symrfmt` and `findsym`.

REVISION HISTORY:

```
Created May 2007 (JKD)
Fixed problem with improper rotations, February 2008 (L. Nordstrom,
F. Bultmark and F. Cricchio)
```

---

## 7.198 symrvfir (Source File: symrvfir.f90)

subroutine symrvfir(ngv,rvfir) *USES:*

```
use modmain
```

*INPUT/OUTPUT PARAMETERS:*

```
ngv   : number of G-vectors to be used for the Fourier space rotation
       (in,integer)
rvfir : real interstitial vector function (inout,real(ngtot,ndmag))
```

DESCRIPTION:

Symmetrises a real interstitial vector function. See routines `symrvf` and `symrfir` for details.

REVISION HISTORY:

```
Created July 2007 (JKD)
```

---

## 7.199 symvect (Source File: symvect.f90)

### INTERFACE:

```
subroutine symvect(tspin,vc)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
  tspin : .true. if the global spin rotations should be used instead of the  
          spatial rotations (in,logical)  
  vc    : vectors in Cartesian coordinates for all atoms (in,real(3,natmtot))
```

### DESCRIPTION:

Symmetrises a 3-vector at each atomic site by rotating and averaging over equivalent atoms. Depending on `tspin`, either the spatial or spin part of the crystal symmetries are used.

### REVISION HISTORY:

```
  Created June 2004 (JKD)  
  Modified for spin rotations, May 2008 (JKD)
```

---

## 7.200 tensmomengy (Source File: tensmomengy.f90)

### INTERFACE:

```
subroutine tensmomengy(is,l,k1,p,r,edir,exch)
```

### USES:

```
use modldapu
```

### INPUT/OUTPUT PARAMETERS:

```
  is   : species type (in,integer)  
  l    : angular momentum (in,integer)  
  k1   : k-index of tensor moment (in,integer)  
  p    : p-index of tensor moment (in,integer)  
  r    : r-index of tensor moment (in,integer)  
  edir : direct energy (out,real)  
  exch : exchange energy (out,real)
```

### DESCRIPTION:

Decomposes the LDA+U Hartree-Fock energy, direct and exchange term, in tensor moments, see *Phys. Rev. B* **80**, 035121 (2009).

### REVISION HISTORY:

```
  Created April 2008 (F.Cricchio and L.Nordstrom)
```

---

## 7.201 tensmom (Source File: tensmom.f90)

### INTERFACE:

```
subroutine tensmom(fnum)
```

### USES:

```
use modmain  
use modmpi  
use modldapu  
use modtest
```

### DESCRIPTION:

Decompose the density matrix and the LDA+U Hartree-Fock energy in tensor moments components, see *Phys. Rev. B* **80**, 035121 (2009).

### REVISION HISTORY:

Created April 2008 (F.Cricchio and L.Nordstrom)

---

## 7.202 vecfbz (Source File: vecfbz.f90)

### INTERFACE:

```
subroutine vecfbz(eps,bvec,vpl)
```

### INPUT/OUTPUT PARAMETERS:

```
eps  : zero component tolerance (in,real)  
bvec : reciprocal lattice vectors (in,real(3,3))  
vpl  : input vector in lattice coordinates (inout,real(3))
```

### DESCRIPTION:

Maps a vector in lattice coordinates to the first Brillouin zone. This is done by first removing its integer components and then adding primitive reciprocal lattice vectors until the shortest vector is found.

### REVISION HISTORY:

Created September 2008 (JKD)

---

## 7.203 vecplot (Source File: vecplot.f90)

### INTERFACE:

```
subroutine vecplot
```

### DESCRIPTION:

Outputs a 2D or 3D vector field for plotting. The vector field can be the magnetisation vector field,  $\mathbf{m}$ ; the exchange-correlation magnetic field,  $\mathbf{B}_{xc}$ ; or the electric field  $\mathbf{E} \equiv -\nabla V_C$ . The magnetisation is obtained from the spin density matrix,  $\rho_{\alpha\beta}$ , by solving

$$\rho_{\alpha\beta}(\mathbf{r}) = \frac{1}{2} (n(\mathbf{r})\delta_{\alpha\beta} + \sigma \cdot \mathbf{m}(\mathbf{r})),$$

where  $n \equiv \text{tr } \rho_{\alpha\beta}$  is the total density. In the case of 2D plots, the magnetisation vectors are still 3D, but are in the coordinate system of the plane.

### REVISION HISTORY:

Created August 2004 (JKD)

Included electric field plots, August 2006 (JKD)

---

## 7.204 wavfmt (Source File: wavfmt.f90)

### INTERFACE:

```
subroutine wavfmt(lrstp,lmax,ias,ngp,apwalm,evecfv,ld,wfmt)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
lrstp : radial step length (in,integer)
lmax  : maximum angular momentum required (in,integer)
ias   : joint atom and species number (in,integer)
ngp   : number of G+p-vectors (in,integer)
apwalm : APW matching coefficients
        (in,complex(ngkmax,apwordmax,lmmxapw,natmtot))
evecfv : first-variational eigenvector (in,complex(nmatmax))
ld     : leading dimension (in,integer)
wfmt   : complex muffin-tin wavefunction passed in as real array
        (out,real(2,ld,*))
```

### DESCRIPTION:

Calculates the first-variational wavefunction in the muffin-tin in terms of a spherical harmonic expansion. For atom  $\alpha$  and a particular  $k$ -point  $\mathbf{p}$ , the  $r$ -dependent  $(l, m)$ -coefficients of the wavefunction for the  $i$ th state are given by

$$\Phi_{alm}^{i\mathbf{p}}(r) = \sum_{\mathbf{G}} b_{\mathbf{G}}^{i\mathbf{p}} \sum_{j=1}^{M_l^\alpha} A_{jlm}^\alpha(\mathbf{G} + \mathbf{p}) u_{jl}^\alpha(r) + \sum_{j=1}^{N^\alpha} b_{(\alpha,j,m)}^{i\mathbf{p}} v_j^\alpha(r) \delta_{l,l_j},$$

where  $b^{i\mathbf{p}}$  is the  $i$ th eigenvector returned from routine `seceqn`;  $A_{jlm}^\alpha(\mathbf{G} + \mathbf{p})$  is the matching coefficient;  $M_l^\alpha$  is the order of the APW;  $u_{jl}^\alpha$  is the APW radial function;  $N^\alpha$  is the number of local-orbitals;  $v_j^\alpha$  is the  $j$ th local-orbital radial function; and  $(\alpha, j, m)$  is a compound index for the location of the local-orbital in the eigenvector. See routines `genapwfr`, `genlofr`, `match` and `seceqn`.

REVISION HISTORY:

Created April 2003 (JKD)  
 Fixed description, October 2004 (C. Brouder)  
 Removed argument `ist`, November 2006 (JKD)

## 7.205 wigner3j (Source File: wigner3j.f90)

INTERFACE:

```
real(8) function wigner3j(j1,j2,j3,m1,m2,m3)
```

INPUT/OUTPUT PARAMETERS:

j1, j2, j3 : angular momentum quantum numbers (in,integer)  
 m1, m2, m3 : magnetic quantum numbers (in,integer)

DESCRIPTION:

Returns the Wigner  $3j$ -symbol. There are many equivalent formulae for the  $3j$ -symbols, the following provides high accuracy for  $j \leq 50$

$$\begin{aligned} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \\ (-1)^{j_1+j_2+m_3} \sqrt{\frac{(j_1+m_1)!(j_2+m_2)!(j_3+m_3)!(j_3-m_3)!(j_1-m_1)!(j_2-m_2)!}{(j_2-j_1+j_3)!(j_1-j_2+j_3)!(j_1+j_2-j_3)!(1+j_1+j_2+j_3)!}} \sum_k (-1)^k \\ \frac{(j_2-j_1+j_3)!(j_1-j_2+j_3)!(j_1+j_2-j_3)!}{(j_3-j_1-m_2+k)!(j_3-j_2+m_1+k)!(j_1+j_2-j_3-k)!k!(j_1-m_1-k)!(j_2+m_2-k)!}, \end{aligned}$$

where the sum is over all integers  $k$  for which the factorials in the summand are non-negative.

REVISION HISTORY:

Created November 2002 (JKD)

## 7.206 wigner6j (Source File: wigner6j.f90)

INTERFACE:

```
real(8) function wigner6j(j1,j2,j3,k1,k2,k3)
```

INPUT/OUTPUT PARAMETERS:

j1, j2, j3 : angular momentum quantum numbers (in,integer)

k1, k2, k3 : angular momentum quantum numbers (in,integer)

DESCRIPTION:

Returns the Wigner  $6j$ -symbol for integer arguments. This is computed using the Racah formula:

$$\left\{ \begin{matrix} j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{matrix} \right\} = \sqrt{\Delta(j_1 j_2 j_3)\Delta(j_1 k_2 k_3)\Delta(k_1 j_2 k_3)\Delta(k_1 k_2 j_3)} \sum_n \frac{(-1)^n (n+1)!}{f(n)},$$

where

$$f(n) = (n - j_1 - j_2 - j_3)! (n - j_1 - k_2 - k_3)! (n - k_1 - j_2 - k_3)! (n - k_1 - k_2 - j_3)! \\ \times (j_1 + j_2 + k_1 + k_2 - n)! (j_2 + j_3 + k_2 + k_3 - n)! (j_1 + j_3 + k_1 + k_3 - n)!$$

and

$$\Delta(a, b, c) = \frac{(a + b - c)! (a - b + c)! (-a + b + c)!}{(a + b + c + 1)!}$$

is the triangle coefficient, and where the sum is over all integers  $n$  for which the factorials in  $f(n)$  have non-negative arguments. The Wigner- $6j$  function is zero unless each triad,  $(j_1 j_2 j_3)$ ,  $(j_1 k_2 k_3)$ ,  $(k_1 j_2 k_3)$  and  $(k_1 k_2 j_3)$ , satisfies the triangle inequalities:

$$|x - y| \leq z \leq x + y,$$

for triad  $(x, y, z)$ . See, for example, A. Messiah, *Quantum Mechanics*, Vol. 2., 1061-1066 (1962).

REVISION HISTORY:

Created August 2009 (JKD)

---

## 7.207 writeefg (Source File: writeefg.f90)

INTERFACE:

```
subroutine writeefg
```

USES:

```
use modmain
```

#### DESCRIPTION:

Computes the electric field gradient (EFG) tensor for each atom,  $\alpha$ , and writes it to the file `EFG.OUT` along with its eigenvalues. The EFG is defined by

$$V_{ij}^{\alpha} \equiv \left. \frac{\partial^2 V_C'(\mathbf{r})}{\partial \mathbf{r}_i \partial \mathbf{r}_j} \right|_{\mathbf{r}=\mathbf{r}_{\alpha}},$$

where  $V_C'$  is the Coulomb potential with the  $l = m = 0$  component removed in each muffin-tin. The derivatives are computed explicitly using the routine `gradrfmt`.

#### REVISION HISTORY:

Created May 2004 (JKD)

Fixed serious problem, November 2006 (JKD)

---

### 7.208 writeengyflu (Source File: writeengyflu.f90)

#### INTERFACE:

```
subroutine writeengyflu
```

#### USES:

```
use modmain  
use modldapu
```

#### DESCRIPTION:

Writes to file the linearisation energies for all radial functions used to calculate the Slater integrals through a Yukawa potential.

#### REVISION HISTORY:

Created July 2008 (Francesco Cricchio)

---

### 7.209 writeeval (Source File: writeeval.f90)

#### INTERFACE:

```
subroutine writeeval
```

#### USES:

```
use modmain
```

#### DESCRIPTION:

Outputs the second-variational eigenvalues and occupation numbers to the file `EIGVAL.OUT`.

#### REVISION HISTORY:

Created June 2003 (JKD)

---

## 7.210 writefermi (Source File: writefermi.f90)

### INTERFACE:

```
subroutine writefermi
```

### USES:

```
use modmain
```

### DESCRIPTION:

Writes the Fermi energy to the file EFERMI.OUT.

### REVISION HISTORY:

Created March 2005 (JKD)

---

## 7.211 writegeom (Source File: writegeom.f90)

### INTERFACE:

```
subroutine writegeom(fnum)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

fnum : file number for writing output (in,integer)

### DESCRIPTION:

Outputs the lattice vectors and atomic positions to file, in a format which may be then used directly in elk.in.

### REVISION HISTORY:

Created January 2004 (JKD)

---

## 7.212 writeiad (Source File: writeiad.f90)

### INTERFACE:

```
subroutine writeiad(fnum)
```

### USES:

use modmain

*INPUT/OUTPUT PARAMETERS:*

fnum : file number for writing output (in,integer)

DESCRIPTION:

Outputs the interatomic distances to file.

REVISION HISTORY:

Created May 2005 (JKD)

---

### 7.213 writeinfo (Source File: writeinfo.f90)

INTERFACE:

```
subroutine writeinfo(fnum)
```

*USES:*

```
use modmain
use modmpi
use modldapu
use modrdm
```

*INPUT/OUTPUT PARAMETERS:*

fnum : unit specifier for INFO.OUT file (in,integer)

DESCRIPTION:

Outputs basic information about the run to the file INFO.OUT. Does not close the file afterwards.

REVISION HISTORY:

Created January 2003 (JKD)  
Updated with LDA+U quantities July 2009 (FC)

---

### 7.214 writekpts (Source File: writekpts.f90)

INTERFACE:

```
subroutine writekpts
```

*USES:*

```
use modmain
```

DESCRIPTION:

Writes the  $k$ -points in lattice coordinates, weights and number of  $\mathbf{G} + \mathbf{k}$ -vectors to the file KPOINTS.OUT.

REVISION HISTORY:

Created June 2003 (JKD)

---

### 7.215 writelinen (Source File: writelinen.f90)

INTERFACE:

```
subroutine writelinen
```

*USES:*

```
use modmain
```

DESCRIPTION:

Writes the linearisation energies for all APW and local-orbital functions to the file LINENGY.OUT.

REVISION HISTORY:

Created February 2004 (JKD)

---

### 7.216 writepmat (Source File: writepmat.f90)

INTERFACE:

```
subroutine writepmat
```

*USES:*

```
use modmain
```

```
use modmpi
```

DESCRIPTION:

Calculates the momentum matrix elements using routine `genpmat` and writes them to direct access file PMAT.OUT.

REVISION HISTORY:

Created November 2003 (Sharma)

---

## 7.217 writestate (Source File: writestate.f90)

### INTERFACE:

```
subroutine writestate
```

### USES:

```
use modmain  
use modldapu
```

### DESCRIPTION:

Writes the charge density, potentials and other relevant variables to the file `STATE.OUT`. Note to developers: changes to the way the variables are written should be mirrored in `readstate`.

### REVISION HISTORY:

Created May 2003 (JKD)

---

## 7.218 writesym (Source File: writesym.f90)

### INTERFACE:

```
subroutine writesym
```

### USES:

```
use modmain
```

### DESCRIPTION:

Outputs the Bravais, crystal and site symmetry matrices to files `SYMLAT.OUT`, `SYMCRYST.OUT` and `SYMSITE.OUT`, respectively. Also writes out equivalent atoms and related crystal symmetries to `EQATOMS.OUT`.

### REVISION HISTORY:

Created October 2002 (JKD)

---

## 7.219 writevnljji (Source File: writevnljji.f90)

### INTERFACE:

```
subroutine writevnljji
```

### USES:

```
use modmain
use modmpi
```

DESCRIPTION:

Generates non-local Coulomb matrix elements of the type  $(i - jj - i)$  and outputs them to the file VNLIJJI.OUT.

REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.220 writevnljkk (Source File: writevnljkk.f90)

INTERFACE:

```
subroutine writevnljkk
```

USES:

```
use modmain
use modmpi
```

DESCRIPTION:

Generates non-local Coulomb matrix elements of the type  $(i - jj - k)$  and outputs them to the file VNLIJJK.OUT. Also writes the real diagonal of this matrix,  $(i - jj - i)$ , to VNLIJJI.OUT.

REVISION HISTORY:

Created 2008 (Sharma)

---

### 7.221 writewiq2 (Source File: writewiq2.f90)

INTERFACE:

```
subroutine writewiq2
```

USES:

```
use modmain
```

DESCRIPTION:

Outputs the integrals of  $1/q^2$  in the small parallelepiped around each  $q$ -point to the file WIQ2.OUT. Note that the integrals are calculated after the  $q$ -point has been mapped to the first Brillouin zone. See routine genwiq2.

REVISION HISTORY:

Created June 2005 (JKD)

---

## 7.222 xc\_am05 (Source File: xc\_am05.f90)

### INTERFACE:

```
subroutine xc_am05(n,rho,grho,g2rho,g3rho,ex,ec,vx,vc)
```

### INPUT/OUTPUT PARAMETERS:

```
  n      : number of density points (in,integer)
  rho    : charge density (in,real(n))
  grho   : |grad rho| (in,real(n))
  g2rho  : grad^2 rho (in,real(n))
  g3rho  : (grad rho).(grad |grad rho|) (in,real(n))
  ex     : exchange energy density (out,real(n))
  ec     : correlation energy density (out,real(n))
  vx     : spin-unpolarised exchange potential (out,real(n))
  vc     : spin-unpolarised correlation potential (out,real(n))
```

### DESCRIPTION:

Spin-unpolarised exchange-correlation potential and energy functional of R. Armiento and A. E. Mattsson, *Phys. Rev. B* **72**, 085108 (2005).

### REVISION HISTORY:

Created April 2005 (RAR); based on xc\_pbe

---

## 7.223 xc\_am05\_point (Source File: xc\_am05.f90)

### INTERFACE:

```
subroutine xc_am05_point(rho,s,u,v,ex,ec,vx,vc,pot)
```

### INPUT/OUTPUT PARAMETERS:

```
  rho : electron density (in,real)
  s   : gradient of n / (2 kF n)
  u   : grad n * grad | grad n | / (n**2 (2 kF)**3)
  v   : laplacian of density / (n**2 (2.d0*kf)**3)
  ex  : exchange energy density (out,real)
  ec  : correlation energy density (out,real)
  vx  : spin-unpolarised exchange potential (out,real)
  vc  : spin-unpolarised correlation potential (out,real)
```

### DESCRIPTION:

Calculate the spin-unpolarised exchange-correlation potential and energy for the Armiento-Mattsson 05 functional for a single point.

### REVISION HISTORY:

Created April 2005 (RAR)

---

## 7.224 xc\_am05\_ldax (Source File: xc\_am05.f90)

### INTERFACE:

```
subroutine xc_am05_ldax(n,ex,vx)
```

### INPUT/OUTPUT PARAMETERS:

```
  n : electron density (in,real)
  ex : exchange energy per electron (out,real)
  vx : exchange potential (out,real)
```

### DESCRIPTION:

Local density approximation exchange.

### REVISION HISTORY:

Created April 2005 (RAR)

---

## 7.225 xc\_am05\_ldapwc (Source File: xc\_am05.f90)

### INTERFACE:

```
subroutine xc_am05_ldapwc(n,ec,vc)
```

### INPUT/OUTPUT PARAMETERS:

```
  n : electron density (in,real)
  ec : correlation energy per electron (out,real)
  vc : correlation potential (out,real)
```

### DESCRIPTION:

Correlation energy and potential of the Perdew-Wang parameterisation of the Ceperley-Alder electron gas *Phys. Rev. B* **45**, 13244 (1992) and *Phys. Rev. Lett.* **45**, 566 (1980). This is a clean-room implementation from paper.

### REVISION HISTORY:

Created April 2005 (RAR)

---

## 7.226 xc\_am05\_labertw (Source File: xc\_am05.f90)

### INTERFACE:

```
subroutine xc_am05_labertw(z,val)
```

#### INPUT/OUTPUT PARAMETERS:

z : function argument (in,real)  
val : value of lambert W function of z (out,real)

#### DESCRIPTION:

Lambert  $W$ -function using the method of Corless, Gonnet, Hare, Jeffrey and Knuth, *Adv. Comp. Math.* **5**, 329 (1996). The approach is based loosely on that in GNU Octave by N. N. Schraudolph, but this implementation is for real values and the principal branch only.

#### REVISION HISTORY:

Created April 2005 (RAR)

---

### 7.227 xc\_pbe (Source File: xc\_pbe.f90)

#### INTERFACE:

```
subroutine xc_pbe(n,kappa,mu,beta,rhoup,rhodn,grho,gup,gdn,g2up,g2dn,g3rho, &  
g3up,g3dn,ex,ec,vxup,vxdn,vcup,vcdn)
```

#### INPUT/OUTPUT PARAMETERS:

n : number of density points (in,integer)  
kappa : parameter for large-gradient limit (in,real)  
mu : gradient expansion coefficient (in,real)  
beta : gradient expansion coefficient (in,real)  
rhoup : spin-up charge density (in,real(n))  
rhodn : spin-down charge density (in,real(n))  
grho : |grad rho| (in,real(n))  
gup : |grad rhoup| (in,real(n))  
gdn : |grad rhodn| (in,real(n))  
g2up : grad<sup>2</sup> rhoup (in,real(n))  
g2dn : grad<sup>2</sup> rhodn (in,real(n))  
g3rho : (grad rho).(grad |grad rho|) (in,real(n))  
g3up : (grad rhoup).(grad |grad rhoup|) (in,real(n))  
g3dn : (grad rhodn).(grad |grad rhodn|) (in,real(n))  
ex : exchange energy density (out,real(n))  
ec : correlation energy density (out,real(n))  
vxup : spin-up exchange potential (out,real(n))  
vxdn : spin-down exchange potential (out,real(n))  
vcup : spin-up correlation potential (out,real(n))  
vcdn : spin-down correlation potential (out,real(n))

#### DESCRIPTION:

Spin-polarised exchange-correlation potential and energy of the generalised gradient approximation functional of J. P. Perdew, K. Burke and M. Ernzerhof *Phys. Rev. Lett.* **77**,

3865 (1996) and **78**, 1396(E) (1997). The parameter  $\kappa$ , which controls the large-gradient limit, can be set to 0.804 or 1.245 corresponding to the value in the original article or the revised version of Y. Zhang and W. Yang, *Phys. Rev. Lett.* **80**, 890 (1998).

REVISION HISTORY:

Modified routines written by K. Burke, October 2004 (JKD)

---

## 7.228 xc\_pwca (Source File: xc\_pwca.f90)

INTERFACE:

```
subroutine xc_pwca(n,rhoup,rhodn,ex,ec,vxup,vxdn,vcup,vcdn)
```

INPUT/OUTPUT PARAMETERS:

```
  n      : number of density points (in,integer)
  rhoup  : spin-up charge density (in,real(n))
  rhodn  : spin-down charge density (in,real(n))
  ex     : exchange energy density (out,real(n))
  ec     : correlation energy density (out,real(n))
  vxup   : spin-up exchange potential (out,real(n))
  vxdn   : spin-down exchange potential (out,real(n))
  vcup   : spin-up correlation potential (out,real(n))
  vcdn   : spin-down correlation potential (out,real(n))
```

DESCRIPTION:

Spin-polarised exchange-correlation potential and energy of the Perdew-Wang parameterisation of the Ceperley-Alder electron gas: *Phys. Rev. B* **45**, 13244 (1992) and *Phys. Rev. Lett.* **45**, 566 (1980).

REVISION HISTORY:

Created January 2004 (JKD)  
Rewrote, October 2011 (JKD)

---

## 7.229 xc\_pzca (Source File: xc\_pzca.f90)

INTERFACE:

```
subroutine xc_pzca(n,rho,ex,ec,vx,vc)
```

INPUT/OUTPUT PARAMETERS:

n : number of density points (in,integer)  
rho : charge density (in,real(n))  
ex : exchange energy density (out,real(n))  
ec : correlation energy density (out,real(n))  
vx : exchange potential (out,real(n))  
vc : correlation potential (out,real(n))

DESCRIPTION:

Spin-unpolarised exchange-correlation potential and energy of the Perdew-Zunger parameterisation of Ceperley-Alder electron gas: *Phys. Rev. B* **23**, 5048 (1981) and *Phys. Rev. Lett.* **45**, 566 (1980).

REVISION HISTORY:

Created October 2002 (JKD)

---

### 7.230 xc\_vbh (Source File: xc\_vbh.f90)

INTERFACE:

```
subroutine xc_vbh(n,rhoup,rhodn,ex,ec,vxup,vxdn,vcup,vcdn)
```

INPUT/OUTPUT PARAMETERS:

n : number of density points (in,integer)  
rhoup : spin-up charge density (in,real(n))  
rhodn : spin-down charge density (in,real(n))  
ex : exchange energy density (out,real(n))  
ec : correlation energy density (out,real(n))  
vxup : spin-up exchange potential (out,real(n))  
vxdn : spin-down exchange potential (out,real(n))  
vcup : spin-up correlation potential (out,real(n))  
vcdn : spin-down correlation potential (out,real(n))

DESCRIPTION:

Spin-polarised exchange-correlation potential and energy functional of von Barth and Hedin: *J. Phys. C* **5**, 1629 (1972). Note that the implementation is in Rydbergs in order to follow the paper step by step, at the end the potential and energy are converted to Hartree.

REVISION HISTORY:

Created September 2007 (F. Cricchio)

---

## 7.231 xc\_xalpha (Source File: xc\_xalpha.f90)

### INTERFACE:

```
subroutine xc_xalpha(n,rho,exc,vxc)
```

### INPUT/OUTPUT PARAMETERS:

```
  n   : number of density points (in,integer)
  rho : charge density (in,real(n))
  exc : exchange-correlation energy density (out,real(n))
  vxc : exchange-correlation potential (out,real(n))
```

### DESCRIPTION:

$X_\alpha$  approximation to the exchange-correlation potential and energy density. See J. C. Slater, *Phys. Rev.* **81**, 385 (1951).

### REVISION HISTORY:

Modified an ABINIT routine, September 2006 (JKD)

---

## 7.232 ylmrot (Source File: ylmrot.f90)

### INTERFACE:

```
subroutine ylmrot(p,alpha,beta,gamma,lmax,ld,d)
```

### INPUT/OUTPUT PARAMETERS:

```
  p      : if p=-1 then the rotation matrix is improper (in,integer)
  alpha  : first Euler angle (in,real)
  beta   : second Euler angle (in,real)
  gamma  : third Euler angle (in,real)
  lmax   : maximum angular momentum (in,integer)
  ld     : leading dimension (in,integer)
  d      : complex spherical harmonic rotation matrix (out,complex(ld,*))
```

### DESCRIPTION:

Returns the rotation matrix in the basis of complex spherical harmonics given the three Euler angles,  $(\alpha, \beta, \gamma)$ , and the parity,  $p$ , of the rotation. The matrix is given by the formula

$$D_{m_1 m_2}^l(\alpha, \beta, \gamma) = d_{m_1 m_2}^l(\beta) e^{-i(m_1 \alpha + m_2 \gamma)},$$

where  $d$  is the rotation matrix about the  $y$ -axis. For improper rotations, i.e. those which are a combination of a rotation and inversion,  $D$  is modified with  $D_{m_1 m_2}^l \rightarrow (-1)^l D_{m_1 m_2}^l$ . See the routines `euler` and `ylmroty`.

### REVISION HISTORY:

Created December 2008 (JKD)

---

### 7.233 ylmroty (Source File: ylmroty.f90)

#### INTERFACE:

```
subroutine ylmroty(beta,lmax,ld,dy)
```

#### INPUT/OUTPUT PARAMETERS:

```
beta : rotation angle about y-axis (in,real)
lmax : maximum angular momentum (in,integer)
ld   : leading dimension (in,integer)
dy   : rotation matrix for complex spherical harmonics (out,real(ld,*))
```

#### DESCRIPTION:

Returns the rotation matrix in the basis of complex spherical harmonics for a rotation of angle  $\beta$  about the  $y$ -axis. This matrix is real and is given by the formula

$$d_{m_1 m_2}^l(\beta) = [(l + m_1)!(l - m_1)!(l + m_2)!(l - m_2)!]^{1/2} \\ \times \sum_k (-1)^k \frac{\left(\cos \frac{\beta}{2}\right)^{2(l-k)-m_2+m_1} \left(\sin \frac{\beta}{2}\right)^{2k+m_2-m_1}}{k!(l + m_1 - k)!(l - m_2 - k)!(m_2 - m_1 + k)!},$$

where  $k$  runs through all integer values for which the factorials exist.

#### REVISION HISTORY:

Created December 2008 (JKD)

---

### 7.234 z2mctm (Source File: z2mctm.f90)

#### INTERFACE:

```
subroutine z2mctm(a,b,c)
```

#### INPUT/OUTPUT PARAMETERS:

```
a   : input matrix 1 (in,complex(2,2))
b   : input matrix 2 (in,complex(2,2))
c   : output matrix (out,complex(2,2))
```

#### DESCRIPTION:

Multiplies the conjugate transpose of one complex  $2 \times 2$  matrix with another. Note that the output matrix cannot be one of the input matrices.

#### REVISION HISTORY:

Created October 2007 (JKD)

---

### 7.235 z2mmct (Source File: z2mmct.f90)

#### INTERFACE:

```
subroutine z2mmct(a,b,c)
```

#### INPUT/OUTPUT PARAMETERS:

```
  a  : input matrix 1 (in,complex(2,2))  
  b  : input matrix 2 (in,complex(2,2))  
  c  : output matrix (out,complex(2,2))
```

#### DESCRIPTION:

Multiplies a  $2 \times 2$  matrix with the conjugate transpose of another. Note that the output matrix cannot be one of the input matrices.

#### REVISION HISTORY:

Created October 2007 (JKD)

---

### 7.236 z2mm (Source File: z2mm.f90)

#### INTERFACE:

```
subroutine z2mm(a,b,c)
```

#### INPUT/OUTPUT PARAMETERS:

```
  a  : input matrix 1 (in,complex(2,2))  
  b  : input matrix 2 (in,complex(2,2))  
  c  : output matrix (out,complex(2,2))
```

#### DESCRIPTION:

Multiplies two complex  $2 \times 2$  matrices. Note that the output matrix cannot be one of the input matrices.

#### REVISION HISTORY:

Created October 2007 (JKD)

---

### 7.237 zbessela (Source File: zbessela.f90)

#### INTERFACE:

```
subroutine zbessela(lmax,x,a)
```

*INPUT/OUTPUT PARAMETERS:*

lmax : maximum order of Bessel function (in,integer)  
x : real argument (in,real)  
a : array of returned values (out,real(0:lmax))

DESCRIPTION:

Computes variations of the spherical Bessel function,  $a_l(x) = i^l j_l(ix)$ , for real argument  $x$  and  $l = 0, 1, \dots, l_{\max}$ . The recursion relation

$$j_{l+1}(x) = \frac{2l+1}{x} j_l(x) + j_{l-1}(x)$$

is used upwards. For starting values there are

$$a_0(x) = \frac{\sinh(x)}{x}; \quad a_1(x) = \frac{a_0(x) - \cosh(x)}{x}$$

. For  $x \ll 1$  the asymptotic forms

$$a_l(x) \approx \frac{(-x)^l}{(2l+1)!!}$$

are used.

REVISION HISTORY:

Created April 2008 from sbessel routine (Lars Nordstrom)

---

## 7.238 zbesselb (Source File: zbesselb.f90)

INTERFACE:

```
subroutine zbesselb(lmax,x,b)
```

*INPUT/OUTPUT PARAMETERS:*

lmax : maximum order of Bessel function (in,integer)  
x : real argument (in,real)  
b : array of returned values (out,real(0:lmax))

DESCRIPTION:

Computes variations of the spherical Bessel function  $b_l(x) = i^l h_l^{(1)}(ix)$ , for real argument  $x$  and  $l = 0, 1, \dots, l_{\max}$ . The recursion relation

$$j_{l+1}(x) = \frac{2l+1}{x} j_l(x) + j_{l-1}(x)$$

is used upwards. For starting values there are

$$b_0(x) = -\frac{e^{-x}}{x}; \quad b_1(x) = b_0(x) \left\{ 1 + \frac{1}{x} \right\}.$$

For  $x \ll 1$  the asymptotic forms

$$b_l(x) \approx \frac{-(2l-1)!!}{(-x)^{l+1}}$$

are used.

#### REVISION HISTORY:

Created April 2008 from sbessel routine (Lars Nordstrom)

---

### 7.239 zfftifc (Source File: zfftifc.f90)

#### INTERFACE:

```
subroutine zfftifc(nd,n,sgn,z)
```

#### INPUT/OUTPUT PARAMETERS:

```
nd   : number of dimensions (in,integer)
n    : grid sizes (in,integer(nd))
sgn  : FFT direction, -1: forward; 1: backward (in,integer)
z    : array to transform (inout,complex(n(1)*n(2)*...*n(nd)))
```

#### DESCRIPTION:

Interface to the double-precision complex fast Fourier transform routine. This is to allow machine-optimised routines to be used without affecting the rest of the code. See routine `nfftifc`.

#### REVISION HISTORY:

Created October 2002 (JKD)

---

### 7.240 zfinp (Source File: zfinp.f90)

#### INTERFACE:

```
complex(8) function zfinp(tsh,zfmt1,zfmt2,zfir1,zfir2)
```

#### USES:

```
use modmain
```

#### INPUT/OUTPUT PARAMETERS:

```

tsh   : .true. if the muffin-tin functions are in spherical harmonics
       (in,logical)
zfmt1 : first complex function in spherical harmonics/coordinates for all
       muffin-tins (in,complex(lmaxvr,nrcmtmax,natmtot))
zfmt2 : second complex function in spherical harmonics/coordinates for all
       muffin-tins (in,complex(lmaxvr,nrcmtmax,natmtot))
zfir1 : first complex interstitial function in real-space
       (in,complex(ngtot))
zfir2 : second complex interstitial function in real-space
       (in,complex(ngtot))

```

DESCRIPTION:

Calculates the inner product of two complex functions over the entire unit cell. The muffin-tin functions should be stored on the coarse radial grid and have angular momentum cut-off `lmaxvr`. In the interstitial region, the integrand is multiplied with the characteristic function, to remove the contribution from the muffin-tin. See routines `zfmtinp` and `gencfun`.

REVISION HISTORY:

Created July 2004 (Sharma)

---

## 7.241 `zflmconj` (Source File: `zflmconj.f90`)

INTERFACE:

```
subroutine zflmconj(lmax,zflm1,zflm2)
```

*INPUT/OUTPUT PARAMETERS:*

```

lmax  : maximum angular momentum (in,integer)
zflm1 : coefficients of input complex spherical harmonic expansion
       (in,complex((lmax+1)**2))
zflm2 : coefficients of output complex spherical harmonic expansion
       (out,complex((lmax+1)**2))

```

DESCRIPTION:

Returns the complex conjugate of a function expanded in spherical harmonics. In other words, given the input function coefficients  $z_{lm}$ , the routine returns  $z'_{lm} = (-1)^m z_{l-m}^*$  so that

$$\sum_{lm} z'_{lm} Y_{lm}(\theta, \phi) = \left( \sum_{lm} z_{lm} Y_{lm}(\theta, \phi) \right)^*$$

for all  $(\theta, \phi)$ .

REVISION HISTORY:

Created April 2004 (JKD)

---

## 7.242 zfmtinp (Source File: zfmtinp.f90)

### INTERFACE:

```
complex(8) function zfmtinp(tsh,nr,nri,r,zfmt1,zfmt2)
```

### USES:

```
use modmain
```

### INPUT/OUTPUT PARAMETERS:

```
tsh  : .true. if the functions are in spherical harmonics (in,logical)
nr   : number of radial mesh points (in,integer)
nri  : number of points on the inner part of the muffin-tin (in,integer)
r    : radial mesh (in,real(nr))
zfmt1 : first complex muffin-tin function in spherical harmonics or
       : coordinates (in,complex(lmmaxvr,nr))
zfmt2 : second complex muffin-tin function in spherical harmonics or
       : coordinates (in,complex(lmmaxvr,nr))
```

### DESCRIPTION:

Calculates the inner product of two complex functions in the muffin-tin. In other words, given two complex functions of the form

$$f(\mathbf{r}) = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l f_{lm}(r) Y_{lm}(\hat{\mathbf{r}}),$$

the function returns

$$I = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l \int f_{lm}^{1*}(r) f_{lm}^2(r) r^2 dr .$$

Note that if `tsh` is `.false.` the functions are in spherical coordinates rather than spherical harmonics. In this case  $I$  is multiplied by  $4\pi/(l_{\max} + 1)^2$ .

### REVISION HISTORY:

```
Created November 2003 (Sharma)
Modified, September 2013 (JKD)
```

---

## 7.243 zftrf (Source File: zftrf.f90)

### INTERFACE:

```
subroutine zftrf(npv,ivp,vpc,rfmt,rfir,zfp)
```

### USES:

```
use modmain
```

#### INPUT/OUTPUT PARAMETERS:

npv : number of P-vectors (in,integer)  
ivp : integer coordinates of the P-vectors (in,integer(3,npv))  
vpc : P-vectors in Cartesian coordinates (in,real(3,npv))  
rfmt : real muffin-tin function (in,real(lmmaxvr,nrmtmax,natmtot))  
rfir : real interstitial function (in,real(ngtot))  
zfp : Fourier expansion coefficients of the real-space function  
(out,complex(npv))

#### DESCRIPTION:

Given a real function periodic in the unit cell,  $f(\mathbf{r})$ , this routine calculates its complex Fourier expansion coefficients:

$$f(\mathbf{P}) = \frac{1}{\Omega} \int d^3r f(\mathbf{r}) \tilde{\Theta}(\mathbf{r}) e^{-i\mathbf{P}\cdot\mathbf{r}} + \frac{4\pi}{\Omega} \sum_{\alpha} e^{-i\mathbf{P}\cdot\mathbf{R}_{\alpha}} \sum_{lm} (-i)^l Y_{lm}(\hat{\mathbf{P}}) \int_0^{R_{\alpha}} dr r^2 j_l(|\mathbf{P}|r) f_{lm}^{\alpha}(r),$$

where  $\tilde{\Theta}$  is the smooth characteristic function of the interstitial region,  $\Omega$  is the unit cell volume and  $R_{\alpha}$  is the muffin-tin radius of atom  $\alpha$ .

#### REVISION HISTORY:

Created July 2010 (Alexey I. Baranov)  
Modified, November 2010 (JKD)

---

### 7.244 zpotclmt (Source File: zpotclmt.f90)

#### INTERFACE:

```
subroutine zpotclmt(nr,nri,r,zrhomt,zvclmt)
```

#### USES:

```
use modmain
```

#### INPUT/OUTPUT PARAMETERS:

nr : number of radial mesh points (in,integer)  
nri : number of points on inner part of muffin-tin (in,integer)  
r : radial mesh (in,real(nr))  
zrhomt : muffin-tin charge density (in,complex(lmmaxvr,nr))  
zvclmt : muffin-tin Coulomb potential (out,complex(lmmaxvr,nr))

#### DESCRIPTION:

Solves the Poisson equation for the charge density contained in an isolated muffin-tin using the Green's function approach. In other words, the spherical harmonic expansion of the Coulomb potential,  $V_{lm}$ , is obtained from the density expansion,  $\rho_{lm}$ , by

$$V_{lm}(r) = \frac{4\pi}{2l+1} \left( \frac{1}{r^{l+1}} \int_0^r \rho_{lm}(r') r'^{l+2} dr' + r^l \int_r^R \frac{\rho_{lm}(r')}{r'^{l-1}} dr' \right)$$

where  $R$  is the muffin-tin radius.

#### REVISION HISTORY:

Created April 2003 (JKD)

---

### 7.245 zpotcoul (Source File: zpotcoul.f90)

#### INTERFACE:

```
subroutine zpotcoul(nr,nri,ld1,r,igp0,gpc,jlgpr,ylmgp,sfacgp,zrhoir,ld2, &
  zvclmt,zvclir,zrho0)
```

#### USES:

```
use modmain
use modphonon
```

#### INPUT/OUTPUT PARAMETERS:

```
nr      : number of radial points for each species (in,integer(nspecies))
nri     : number of points on inner part of muffin-tin (in,integer(nspecies))
ld1     : leading dimension (in,integer)
r       : radial mesh for each species (in,real(ld1,nspecies))
igp0    : index of the shortest G+p-vector (in,integer)
gpc     : G+p-vector lengths (in,real(ngvec))
jlgpr   : spherical Bessel functions for every G+p-vector and muffin-tin
          radius (in,real(0:lnpsd,ngvec,nspecies))
ylmgp   : spherical harmonics of the G+p-vectors (in,complex(lmmaxvr,ngvec))
sfacgp  : structure factors of the G+p-vectors (in,complex(ngvec,natmtot))
zrhoir  : interstitial charge density (in,complex(ngtot))
ld2     : leading dimension (in,integer)
zvclmt  : muffin-tin Coulomb potential, with the contribution from the
          isolated muffin-tin density precalculated and passed in
          (inout,complex(lmmaxvr,ld2,natmtot))
zvclir  : interstitial Coulomb potential (out,complex(ngtot))
zrho0   : G+p=0 term of the pseudocharge density (out,complex)
```

#### DESCRIPTION:

Calculates the Coulomb potential of a complex charge density by solving Poisson's equation. First, the multipole moments of the muffin-tin charge are determined for the  $j$ th atom of the  $i$ th species by

$$q_{ij;lm}^{\text{MT}} = \int_0^{R_i} r^{l+2} \rho_{ij;lm}(r) dr + z_{ij} Y_{00} \delta_{l,0},$$

where  $R_i$  is the muffin-tin radius and  $z_{ij}$  is a point charge located at the atom center (usually the nuclear charge, which should be taken as **negative**). Next, the multipole moments of the continuation of the interstitial density,  $\rho^{\text{I}}$ , into the muffin-tin are found with

$$q_{ij;lm}^{\text{I}} = 4\pi i^l R_i^{l+3} \sum_{\mathbf{G}} \frac{j_{l+1}(GR_i)}{GR_i} \rho^{\text{I}}(\mathbf{G}) \exp(i\mathbf{G} \cdot \mathbf{r}_{ij}) Y_{lm}^*(\hat{\mathbf{G}}),$$

remembering that

$$\lim_{x \rightarrow 0} \frac{j_{l+n}(x)}{x^n} = \frac{1}{(2n+1)!!} \delta_{l,0}$$

should be used for the case  $\mathbf{G} = 0$ . A pseudocharge is now constructed which is equal to the real density in the interstitial region and whose multipoles are the difference between the real and interstitial muffin-tin multipoles. This pseudocharge density is smooth in the sense that it can be expanded in terms of the finite set of  $\mathbf{G}$ -vectors. In each muffin-tin the pseudocharge has the form

$$\rho_{ij}^{\text{P}}(\mathbf{r}) = \rho^{\text{I}}(\mathbf{r} - \mathbf{r}_{ij}) + \sum_{lm} \rho_{ij;lm}^{\text{P}} \frac{1}{R_i^{l+3}} \left(\frac{r}{R_i}\right)^l \left(1 - \frac{r^2}{R_i^2}\right)^{N_i} Y_{lm}(\hat{\mathbf{r}})$$

where

$$\rho_{ij;lm}^{\text{P}} = \frac{(2l + 2N_i + 3)!!}{2_i^N N_i! (2l + 1)!!} (q_{ij;lm}^{\text{MT}} - q_{ij;lm}^{\text{I}})$$

and  $N_i \approx \frac{1}{2} R_i G_{\text{max}}$  is generally a good choice. The pseudocharge in reciprocal space is given by

$$\rho^{\text{P}}(\mathbf{G}) = \rho^{\text{I}}(\mathbf{G}) + \sum_{ij;lm} 2^{N_i} N_i! \frac{4\pi (-i)^l j_{l+N_i+1}(GR_i)}{\Omega R_i^l (GR_i)^{N_i+1}} \rho_{ij;lm}^{\text{P}} \exp(-i\mathbf{G} \cdot \mathbf{r}_{ij}) Y_{lm}(\hat{\mathbf{G}})$$

which may be used for solving Poisson's equation directly

$$V^{\text{P}}(\mathbf{G}) = \begin{cases} 4\pi \frac{\rho^{\text{P}}(\mathbf{G})}{G^2} & G > 0 \\ 0 & G = 0 \end{cases}.$$

The usual Green's function approach is then employed to determine the potential in the muffin-tin sphere due to charge in the sphere. In other words

$$V_{ij;lm}^{\text{MT}}(r) = \frac{4\pi}{2l+1} \left( \frac{1}{r^{l+1}} \int_0^r \rho_{ij;lm}^{\text{MT}}(r') r'^{l+2} dr' + r^l \int_r^{R_i} \frac{\rho_{ij;lm}^{\text{MT}}(r')}{r'^{l-1}} dr' \right) + \frac{1}{Y_{00}} \frac{z_{ij}}{r} \delta_{l,0}$$

where the last term is the monopole arising from the point charge. All that remains is to add the homogenous solution of Poisson's equation,

$$V_{ij}^{\text{H}}(\mathbf{r}) = \sum_{lm} V_{ij;lm}^{\text{H}} \left(\frac{r}{R_i}\right)^l Y_{lm}(\hat{\mathbf{r}}),$$

to the muffin-tin potential so that it is continuous at the muffin-tin boundary. Therefore the coefficients,  $\rho_{ij;lm}^{\text{H}}$ , are given by

$$V_{ij;lm}^{\text{H}} = 4\pi i^l \sum_{\mathbf{G}} j_l(Gr) V^{\text{P}}(\mathbf{G}) \exp(i\mathbf{G} \cdot \mathbf{r}_{ij}) Y_{lm}^*(\hat{\mathbf{G}}) - V_{ij;lm}^{\text{MT}}(R_i).$$

Finally note that the  $\mathbf{G}$ -vectors passed to the routine can represent vectors with a non-zero offset,  $\mathbf{G} + \mathbf{p}$  say, which is required for calculating Coulomb matrix elements.

REVISION HISTORY:

Created April 2003 (JKD)

## 7.246 ztorflm (Source File: ztorflm.f90)

INTERFACE:

```
subroutine ztorflm(lmax,zflm,rflm)
```

*INPUT/OUTPUT PARAMETERS:*

```
lmax : maximum angular momentum (in,integer)
zflm : coefficients of complex spherical harmonic expansion
      (in,complex((lmax+1)**2))
rflm : coefficients of real spherical harmonic expansion
      (out,real((lmax+1)**2))
```

DESCRIPTION:

Converts a real function,  $z_{lm}$ , expanded in terms of complex spherical harmonics into a real spherical harmonic expansion,  $r_{lm}$ :

$$r_{lm} = \begin{cases} \frac{1}{\sqrt{2}} \Re(z_{lm} + (-1)^m z_{l-m}) & m > 0 \\ \frac{1}{\sqrt{2}} \Im(-z_{lm} + (-1)^m z_{l-m}) & m < 0 \\ \Re(z_{lm}) & m = 0 \end{cases} .$$

See routine `genrlm`.

REVISION HISTORY:

Created April 2003 (JKD)